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Discussion

Comment on "Influence of the Lode parameter and the stress triaxiality on the failure of elasto-plastic porous materials" by K. Danas and P. Ponte Castañeda

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This extended comment is in response to a recent paper by Danas and Ponte Castañeda (2012) which investigates the stress–strain behavior and strain localization of porous metallic materials as a function of the Lode parameter under low triaxality stressing. The topic is of some importance owing to its relevance to recent experimental and theoretical work on the role of the Lode parameter in ductile fracture. Specifically, these authors purport to show that strain localization is essentially simultaneous with void collapse under low triaxiality axisymmetric stressing conditions combining equi-biaxial tension plus a hydrostatic component (corresponding to a Lode parameter, $L = 1$). Here, simulations will be reported for the behavior of an initially spherical void under these stressing conditions which reveal that nothing untoward occurs at collapse when the void faces make contact. The drop in incremental stiffness followed by an abrupt strain localization at collapse predicted by Danas and Ponte Castañeda does not occur and is most likely a consequence of analytical assumptions made by these authors related to the behavior of the voids at collapse.

A growing body of work has highlighted the importance of the Lode parameter, L, in addition to stress triaxiality, T, in shear localization and ductile fracture of isotropic structural alloys (e.g., Bao and Wierzbicki (2004), Nahshon and Hutchinson (2008)), where

$$L = \frac{2\sigma_{II} - \sigma_I - \sigma_{III}}{\sigma_I - \sigma_{III}}, T = \frac{\sigma_{kk}/3}{\sigma_e} = \frac{\sqrt{2}(\sigma_I + \sigma_{II} + \sigma_{III})}{3\sqrt{\left[(\sigma_I - \sigma_{II})^2 + (\sigma_I - \sigma_{III})^2 + (\sigma_{II} - \sigma_{III})^2\right]}} \quad (1)$$

with $\sigma_I \geqslant \sigma_{II} \geqslant \sigma_{III}$ as the principal stresses and σ_e as the Mises effective stress. Recent simulations of strain localization by Barsoum and Faleskog (2011) will be used to provide some background for this comment and to set the stage for the discussion that follows. Fig. 1 plots the effective plastic strain at localization, ε_p^c, as a function of the Lode parameter for four relatively high levels of triaxiality. Barsoum and Faleskog performed a rigorous three dimensional localization analysis. The response of the material within the band of localization is computed using a three dimensional periodic cell model with an initial spherical void. The stress state (specified by L and T) and the effective plastic strain are imposed on the slabs of material outside the band, which is taken to be damage-free. Continuity of displacements and average tractions is imposed across the interfaces between the outside slabs and the band. For each stress state, all possible initial orientations of the band relative to the principal stress axes are considered. The effective plastic strain in Fig. 1 is the strain outside the band at the onset of localization for the initial band orientation that gives the minimum onset strain. No further straining takes place outside the band after the onset of localization, and, thus, the localization strain provides a measure of the macroscopic fracture strain.

The effect of the Lode parameter on the localization strain seen in Fig. 1 indicates that, for any fixed triaxiality, the material is most susceptible to localization for stress states with $L \cong 0$. The states with $L = 0$ correspond to a shear stress combined with a hydrostatic stress state (Nahshon and Hutchinson, 2008). By contrast the largest localization strains are associated with axisymmetric stress states, $L = \pm 1$; $L = -1$ corresponds to uniaxial tension plus a hydrostatic stress state, and $L = 1$ corresponds to equi-biaxial tension plus a hydrostatic stress state. The relatively high triaxiality levels in Fig. 1 ensure that the voids in the Barsoum–Faleskog simulations do not collapse or experience void surface contact.

Danas and Ponte Castañeda (2012) address the Lode parameter dependence of strain localization over a range of triaxiality using analytical representations of void deformation. In particular, these authors analyze the case $L = 1$ under low triaxiality conditions in which voids collapse towards penny shaped cracks with void surface contact. Based on their analysis, they conclude that the trends seen in Fig. 1 change dramatically when void collapse occurs—their results then suggest that states with $L = 1$ give rise to the *lowest* localization strains for all values of L. We believe this conclusion is erroneous and most likely due to incorrect assumptions made in conjunction with void surface contact and its consequences in the authors' analytical approach. To support this assertion we present results in Figs. 2 and 3 from a finite element simulation that fully accounts for void surface contact under axisymmetric stress conditions with $L = 1$. The triaxiality, $T = 0.1$, and material proprieties employed in the simulations in Figs. 2 and 3 are those used by Danas and Ponte Castañeda (2012) in the example in their Fig. 5. The cell model results in Figs. 2 and 3 for a material with initially spherical voids and an initial void volume fraction, $f_0 = 0.01$, reveal that the voids flatten to penny-shaped cracks at an overall effective plastic strain about $\varepsilon_e^p \cong 0.6$, in agreement with the prediction of Danas and Ponte Castañeda (2012). Danas and Ponte Castañeda argue that the radius of each void grows suddenly as the void

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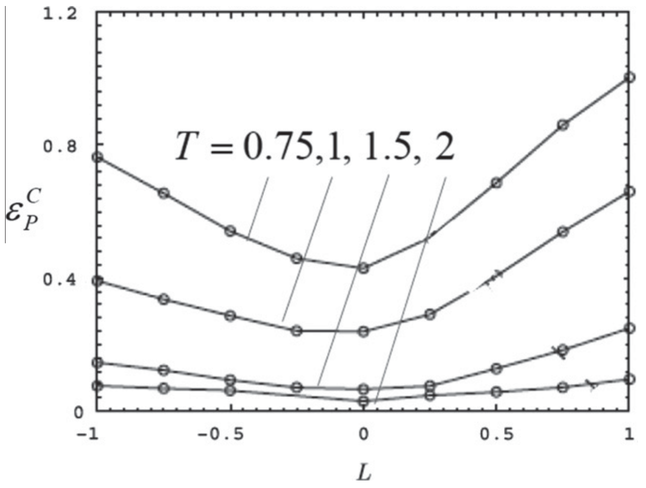

Fig. 1. Effective logarithmic plastic strain at localization as dependent on the Lode parameter, L, for four levels of stress triaxiality, T, adapted from the three dimensional simulations of Barsoum and Faleskog (2011). The base material has an initial tensile yield strain, 0.0025, and strain hardening index 0.1. The initial void volume fraction within the localization band is the same in all the simulations (≈ 0.005) while there is no void damage in the material outside the band. The localization strain is the strain outside the band at the onset of localization.

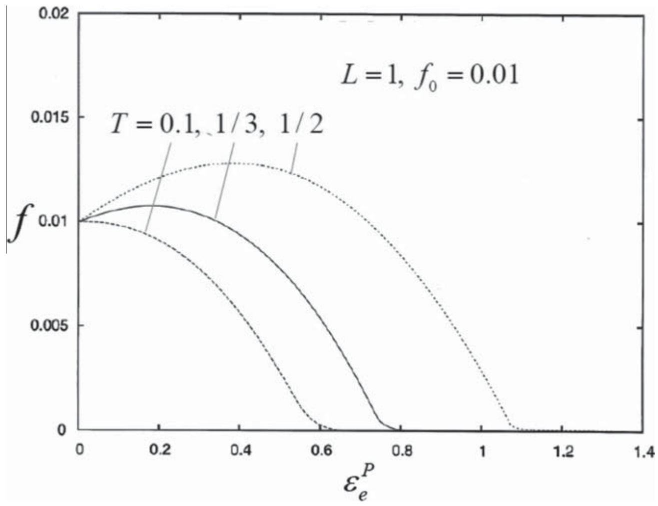

Fig. 2. Simulation for an axisymmetric cell model with an initial spherical void having an initial void volume fraction, $f_0 = 0.01$, an aspect ratio of unity, and applied stresses with $T = 0.1$ and $L = 1$ ($\sigma_I = \sigma_{II}, \sigma_{III}/\sigma_I = -1.308$). Curves for $T = 1/3$ & $1/2$ are also shown. The base material has initial yield stress to modulus ratio, $\sigma_Y/E = 0.001$, and strain hardening index, 0.1. The curve is the relation between the void volume fraction and the overall effective logarithmic plastic strain. For $T = 0.1$, the void flattens and its surfaces first make contact at $\varepsilon_e^P \cong 0.55$; complete collapse of the void with nearly full surface contact occurs at ε_e^P slightly above 0.6.

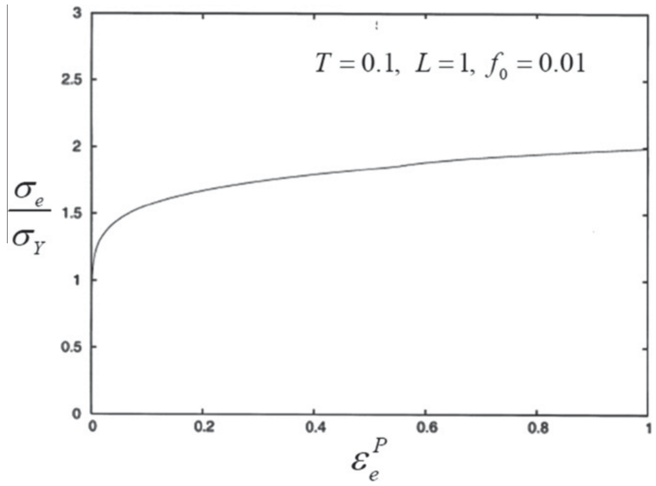

Fig. 3. The overall relation between the effective stress normalize by the initial yield stress and the effective plastic strain for the simulation specified in Fig. 2 with $T = 0.1$ and $L = 1$. The stress–strain curve displays a slight increase in tangent modulus at the point when the void surfaces first make contact at $\varepsilon_e^P \cong 0.55$.

flattens to a penny-shaped crack giving rise to coalescence of the voids. This, they argue, is the source of the low localization strains when $L = 1$ at low triaxiality. We see no evidence for such behavior in our numerical simulations—the radius of the void changes very little as the void flattens to a crack and as the surfaces make contact. Moreover, we observe no drop in the incremental stiffness when the void faces make contact. Instead, the overall stress–strain behavior is even slightly enhanced in a way which would retard or suppress localization when the voids collapse occurs. A slight increase in the tangent modulus of the overall stress–strain curve in Fig. 3 at the strain $\varepsilon_e^P \cong 0.55$ when void surface contact first occurs can be observed, in accord with physical expectations. Curves for $T = 1/3$ & $1/2$ with $L = 1$ are also included in Fig. 2 displaying similar qualitative behavior. At T greater than about 0.6, the void surface contact does not occur, in accord with Danas and Ponte Castañeda (2012).

In summary, the simulations presented here call into question the low localization strains in the range of low triaxiality for stress states with $L = 1$ found by Danas and Ponte Castañeda (2012). In fact, we believe that void collapse with surface contact is likely to produce even larger localization strains for $L = 1$ at low triaxiality, relative to those for other values of the Lode parameter, than seen in the trends in Fig. 1. The issue is an important one because efforts are underway to modify fracture criteria based on a critical effective plastic strain to account for a dependence on the Lode parameter. Accounting for crack surface contact is very important at low stress triaxiality. Recent plane strain analyses for voids in a material subject to simple shear (Tvergaard, 2009; Dahl et al., 2012) have shown that voids collapse to cracks at an early stage and that the average stress continues to increase significantly before plastic flow localization occurs due to other mechanisms. An extension of the localization calculations of Barsoum and Faleskog (2011) into the range of low triaxiality would be most valuable. It is also worth noting that effects such as friction between contacting faces and de-bonded particles also play a role in determining the material constitutive response.

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