1 Background to the Present Study

Cutting by forced shear-off, often called blanking or cropping, is widely employed to cut flat sheets and plates of structural metals having thickness ranging from submillimeter to 10 mm or more. Forced shear-off can also occur in the plugging process in ballistic penetration when a high velocity projectile impacts a plate. The simplest notions of cropping imagine that the process is essentially a shear failure within a localized shear band that is primarily controlled by the shear strength of the material. The present paper continues the line of investigation initiated in a series of papers by Atkins [1–4] who takes the view that shear cracking and material shear toughness play an essential role in the cropping process and in determining the energy required for cropping. The series of Atkins’ papers presents models of cropping with an increasing level of sophistication without recourse to finite element models of finite plastic straining. An early paper by Zhou and Wierzbicki [5] also employs analytical modeling and accounts for tensile fracture as well as shearing in the cropping process, which in their paper and in some of the earlier literature is referred to as blanking.

Following Atkins, the view adopted in this paper is that cropping is a large scale yielding fracture problem dominated by shear. As in the case of the present authors’ earlier work on elastic–plastic crack growth in mode I [6] and mixed mode [7], the present paper exploits the finite element analysis to deal with large plastic strains. The present paper also makes use of recent developments in modeling fracture under shear dominated conditions. The cropping model developed here embeds a cohesive shearing zone, which, in a phenomenological manner, represents the microscopic shear localization and fracture processes, within a finite element model that accounts for the geometric distribution of the large plastic shear strains that occur in the cropping process. The model quantifies the interplay between the microscopic shear failure process and the extensive macroscopic plastic shearing. A systematic study is made of the effects of the microscopic fracture energy and the material stress–strain properties on the cropping force–displacement behavior and the macroscopic cropping energy.

Alternative approaches to studying cropping could be based on analyses that adopt either a critical effective plastic strain as a failure criterion or a constitutive law that incorporates damage and a failure criterion. When carefully calibrated, the critical plastic strain criterion has proved effective in the studies of ballistic plugging of plates by projectiles, e.g., Borvik et al. [8,9] and Nahshon et al. [10]. Xue et al. [11] demonstrated the applicability to ballistic shear-off of the Gurson model [12,13] of void-based damage and failure, extended to account for damage in shear. A distinct advantage of the cohesive zone model adopted in the present paper is that the microscopic strength and fracture energy are well defined parameters and their role in establishing macroscopic behavior clearly emerges.

1.1 An Illustrative Cropping Experiment. Xue et al. [11] carried out carefully designed cropping tests on plates of the steel DH 36 of thickness \( h = 3 \) mm. As illustrated in Fig. 1, the tests provided the response of a hard tool steel cylindrical plunger pushing against a plate clamped tightly outside the plunger radius by a stiff fixture. The nominal shear stress supported by the plate at radius \( R, \) \( F/(2\pi Rh) \), is plotted as a function of the plunger displacement divided by the plate thickness \( \Delta/h \) for one test in Fig. 1(b). The peak nominal shear stress is about 420 MPa at \( \Delta/h \approx 0.25 \). The largest plunger displacement shown is \( \Delta/h \approx 0.55 \). The load drops precipitously at larger displacements such that the macroscopic work of cropping is directly related to the area under the curve in Fig. 1(b). Tests were interrupted at various stages and cross sections of the sheared-off region were opened to viewing by electrodisharge machining (EDM). In addition to intense plastic shearing throughout the region of shear-off, shear cracks are observed. Examples of shear cracks near the corner of the plunger at \( \Delta/h \approx 0.5 \) are seen in Fig. 1(d), where it is noted that some cracks form slightly offset from the line extending the corner of the plunger.

The radius of the plunger in the Xue et al. [11] test is sufficiently large compared to the plate thickness, i.e., \( R/h = 6.3 \), such that it is reasonable to assume that the specific cropping work \( \Gamma_{\text{CROP}} \), with units work/area cut, would be nearly the same as that measured for a long straight cut. The total work done by the plunger in Fig. 1(b) is \( W = \pi z R \phi'_{\text{MAX}} \), where \( \phi'_{\text{MAX}} \) is the maximum nominal shear stress and \( z \) is a dimensionless numerical factor that depends on the parameters of the system. The specific cropping work (per area cut) is

\[
\Gamma_{\text{CROP}} = \frac{W}{\pi Rh} = z(\phi'_{\text{MAX}}) \quad (1)
\]
For the 3 mm thick plate of DH 36 in Fig. 1(b), $(f_Y)_{max}$ = 420 MPa and the area under the load–displacement curve implies $z \approx 1/2$, such that $\Gamma_{CROP} \approx 6 \times 10^3$ Jm$^{-2}$. This value is consistent with the specific work of cropping reported for other metals in the early work of Johnson and Slater [14] discussed in [4,5].

One of the aims of this paper is to relate the specific microscopic work of cropping $\Gamma_{CROP}$ to the specific microscopic work of shear fracture of the material $\Gamma_0$, both of which are measured in units of energy/area, Jm$^{-2}$. Figure 2 presents simulations based on the extended Gurson model [13] for the shear stress–strain behavior accounting for damage in the form of voids with an initial effective volume fraction $f_0$. A pure power-law material is used to describe the undamaged material in the Gurson model with a tensile relation between the true stress and logarithmic strain given by $\sigma = \sigma_R \gamma^\beta$. The extended model accounts for progressive shear damage, due to shear distortion of the voids, in a phenomenological manner such that in shear (with $\gamma$ stress triaxiality) damage grows according to $f = f_0 \gamma^\beta$, with $\gamma$ as the logarithmic shear strain and $k_\omega$ as the shear damage coefficient [13,15,16]. The trends in Figs. 2(a) and 2(b) should only be regarded as qualitative since no effort has been made to include the final coalescence stage of the shear failure process. Moreover, microscopic mechanisms other than the void mechanism may be responsible for softening and shear localization. Nevertheless, the results illustrate the roles of damage $f_0$ and damage growth susceptibility as measured by $k_\omega$ in determining the peak shear stress at which shear localization would begin. This stress–strain behavior has been used to compute the work dissipated in shearing $\gamma$ subsequent to attainment of the peak shear stress in a shear band of thickness $D$. Figure 2(c) presents the specific work dissipated in the shear band per area $\Gamma_0$, normalized by $\sigma_R D$, as a function of the shear damage coefficient for several values of the initial damage. We emphasize that the purpose of Fig. 2 is not to present quantitatively reliable predictions for the shear response or the work dissipated in a shear band but, rather, to suggest trends and to provide rough estimates.

Figure 2(c) indicates that the specific work of shear fracture scales as

$$\Gamma_0 = \beta \sigma_R D \quad (2)$$

where $\beta$ is a factor of order unity depending on the initial damage level and the susceptibility of the damage to shearing as modeled here by the coefficient $k_\omega$. For materials failing by the mechanism of void nucleation, shear distortion, and coalescence, the thickness of the shear localization zone before the final coalescence stage is usually on the order of the spacing between the dominant voids. Thus, for many ductile alloys, $D$ is typically measured in tens of microns. Assume this is so for DH 36, and note also that for this material the tensile stress at a log strain extrapolated to unity is $\sigma_R \approx 1000$ MPa. With $\beta = 2$ in (2), this implies for DH 36 that $\Gamma_0$ is estimated to lie within the range $2 \times 10^3$ Jm$^{-2}$ (for $D = 10\mu m$) to $1 \times 10^3$ Jm$^{-2}$ (for $D = 50\mu m$). The specific macroscopic work of cropping (1) for the 3 mm plate of DH 36 was estimated to be $\Gamma_{CROP} \approx 6 \times 10^3$ Jm$^{-2}$. This comparison suggests that the work of cropping is at least 6 times, and possibly as much at 30 times,
the gap between the surfaces of the cropping tool is and the specific work of fracture in shear in a cohesive zone whose primary parameters are the peak shear stress \( \tau \), initial shear yield stress \( \sigma_y \), initial tensile yield stress \( \sigma_t \), initial shear yield stress \( \tau_y = \sigma_y / \sqrt{3} \), and strain hardening exponent \( N \). In this paper no attempt will be made to account for temperature or rate effects in cropping. The process is modeled as quasi-static and the plate material is taken to be rate independent, excluding any direct relevance to cropping at high temperatures. Elasticity of the cropping tool is also neglected in this study—the surfaces of the cutting tool are taken to be nondeforming. As depicted in Figs. 1(a) and 3(b), the thickness of the plate is \( h \) and the gap between the surfaces of the cropping tool is \( d \). The band of localized shear and shear fracture will be modeled by a cohesive zone whose primary parameters are the peak shear stress \( \tau \) and the specific work of fracture in shear \( \Gamma_\sigma \). Full details of the cohesive zone will be given in the next section, including the maximum shearing displacement across the zone prior to loss of shear strength. The maximum shearing displacement can be expressed in terms of \( \tau \) and \( \Gamma_\sigma \), which are preferred for specifying the cohesive law in the present study.

It is useful to define the following material reference length:

\[
R_s = \frac{1}{\pi(1 - \nu^2)} \frac{E\Gamma_0}{\tau_y^2} \tag{3}
\]

which can be interpreted as the extent of the plastic zone ahead of a mode II (shear) crack in plane strain small scale yielding when subject to the mode II stress intensity factor \( K_{II} = \sqrt{E\Gamma_0/(1 - \nu^2)} \). The only independent material-based length parameter in the present study is \( R_s \). It is important to appreciate that \( R_s \) is not the plastic zone size of a mode II crack with specific microscopic work of fracture \( \Gamma_\sigma \)—that plastic zone size would generally be much larger, i.e., given by (3) with the specific macroscopic work of mode II fracture \( \Gamma_\sigma \) replacing \( \Gamma_0 \).
earlier work on mode I cracking, Tvergaard and Hutchinson [6] introduced the corresponding length for a tensile crack \( R_0 = 1/[3(1 - \nu^2)]EY_0/\sigma_d^2 \), which is \( R_5/9 \). With \( \Gamma_{\text{CROP}} \) as the specific work expended by the cropping plunger (per area of plate cut) for long straight cuts, two dimensionless forms for the specific cropping work in terms of the parameters identified above will be considered

\[
\Gamma_{\text{CROP}} \frac{\tau_y h}{\gamma} = g_1 \left( \frac{h}{R_5} \frac{\bar{\gamma}}{\tau_y} N \min \frac{d}{h}, \frac{\tau_y}{E} \right)
\]

\[
\Gamma_{\text{CROP}} \frac{\tau_y h}{\gamma} = g_2 \left( \frac{h}{R_5} \frac{\bar{\gamma}}{\tau_y} N \min \frac{d}{h}, \frac{\tau_y}{E} \right)
\]

where the dependence on \( \nu \) has not been noted explicitly, and

\[
g_1 = \frac{\Gamma_0}{\tau_y h} g_2 = \pi (1 - \nu^2) \frac{\tau_y R_5}{E} \frac{d}{h} \frac{\tau_y}{E} g_2
\]

Both normalizations will be used to reveal important aspects of the parametric dependencies. For example, it will be seen that the dependence on \( \tau_y \) appears mainly through the first two dimensionless parameters in \( g_2 \) in (5) and not through the fifth dimensionless parameter \( \tau_y/E \), while \( g_1 \) in (4) has a stronger dependence on \( \tau_y/E \) in its list of parameters.

The preferred dimensionless relation relating the cropping force per length \( F \) with units \( \text{Nm}^{-1} = \text{Jm}^{-2} \), and the displacement of the cropping tool through which it works \( \Delta \) is

\[
\frac{F}{\tau_y h} = g_1 \left( \frac{\Delta}{h} \frac{d}{h}, \frac{\tau_y}{E} \right)
\]

By (4), one notes that \( g_1 = \int g_1 d(\Delta/h) \).

### 3 Prescription of the Computational Model

#### 3.1 The Cohesive Zone

As noted above, a cohesive zone model is used to characterize the strength and fracture interface of the plane along which cutting occurs. Because the cutting process is not strictly shearing, a mixed mode traction-separation law is employed in the form introduced in [7] for studying mixed mode interface crack growth. While the tangential displacement \( \delta_t \) across the failure plane is expected to dominate the cutting behavior, the model also accounts for a normal separation \( \delta_n \). Denote the critical value of \( \delta_t \) at which the traction vanishes under strictly shearing as \( \delta_t^\text{crit} \) and similarly denote the critical strictly normal separation by \( \delta_n^\text{crit} \). Under mixed mode conditions, the displacement measure \( \delta = \left( \delta_t/\delta_t^\text{crit} \right)^{1/2} + \left( \delta_n/\delta_n^\text{crit} \right) \) is used such that the traction drops to zero at \( \delta = 1 \). The tractions are derived from a potential function given by

\[
\Phi(\delta_t, \delta_n) = \delta_t^\text{crit} \int_0^1 \tau(\lambda^\text{cr})d\lambda^\text{cr}
\]

where \( \tau(\lambda) \) characterizes the traction-separation relation in shear. Use of the potential results in a work of cohesive failure that is the same for all mixed mode separations, which is given by (8) with \( \lambda = 1 \), i.e.,

\[
\Gamma_0 = \delta_t^\text{crit} \int_0^1 \tau(\lambda^\text{cr})d\lambda^\text{cr}.
\]

The normal and tangential components of the traction acting on the cohesive plane failure are given by

\[
T_n = \frac{\partial \Phi}{\partial \delta_n} = \frac{\tau(\lambda)}{\lambda} \frac{\delta_t^\text{crit}}{\delta_n^\text{crit}}, \quad T_t = \frac{\partial \Phi}{\partial \delta_t} = \frac{\tau(\lambda)}{\lambda} \frac{\delta_t^\text{crit}}{\delta_n^\text{crit}}
\]

In this paper the following piecewise linear traction-separation law is used (see Fig. 3(a)):

\[
\tau(\lambda) = \frac{\lambda}{\lambda_1} \bar{\tau} \quad \text{for} \quad 0 \leq \lambda < \lambda_1
\]

\[
\tau(\lambda) = \frac{1 - \lambda}{1 - \lambda_2} \bar{\tau} \quad \text{for} \quad \lambda_2 < \lambda \leq 1
\]

Here \( \bar{\tau} \) denotes the maximum shear traction sustained by the failure plane under a mode II shear, i.e., \( \delta_n = 0 \). The peak normal traction under mode I separation is \( \bar{\tau} = \delta_t^\text{crit}/\delta_t^\text{crit} \). The specific work of separation per unit area of interface is

\[
\Gamma_0 = \frac{1}{2} \bar{\tau} \delta_t^\text{crit} (1 - \lambda_1 + \lambda_2)
\]

In all the simulations carried out in this paper, the values \( \lambda_1 = 0.15 \) and \( \lambda_2 = 0.5 \) have been used such that \( \Gamma_0 = 0.675\bar{\tau} \).

#### 3.2 The Elastic–Plastic Constitutive Behavior and the Finite Element Model

The cropped material is elastic–plastic, with the elastic modulus and Poisson’s ratio \( E \) and \( \nu \), uniaxial yield stress \( \sigma_y \), and strain hardening exponent \( N \). This material is described by a finite strain generalization of the \( J_2 \)-flow theory [17], with the uniaxial true stress–natural strain curve represented by a piecewise power law

\[
\nu = \begin{cases} 
\sigma/E, & \sigma \leq \sigma_y \\
\sigma_y (\sigma/\sigma_y)^{1/N}, & \sigma > \sigma_y
\end{cases}
\]

A Lagrangian convected coordinate formulation of the field equations is used for the analyses, with a material point identified by the coordinates \( x_i \) in the reference configuration, accounting for finite strains. The contravariant components of the Cauchy stress tensor \( \sigma^{ij} \) and the Kirchhoff stress tensor \( \tau^{ij} \) are related by \( \tau^{ij} = \sqrt{G/\sigma^{ij}} \). The metric tensors in the current and reference configurations are denoted by \( G_{ij} \) and \( g_{ij} \), with determinants \( G \) and \( g \), and the incremental stress–strain relationship is of the form \( k^{ij} = L^{ijk} \eta_k \), where \( L^{ijk} \) are the instantaneous moduli.

The Lagrangian strain tensor is given by

\[
\eta_{ij} = \frac{1}{2} \left( u_{ij} + u_{ji} + u_i^j u_k^k \right)
\]

where \( u_i \) are the displacement components on the reference base vectors and \( (i,j) \) denotes covariant differentiation in the reference frame. Numerical solutions are obtained by a linear incremental solution procedure, based on the principle of virtual work

\[
\int_V \tau^i \delta \eta_{ij} dV + \int_S \{ T_n \delta(\delta_n) + T_t \delta(\delta_t) \} dS = \int_S T^i \delta u_i dA
\]

Here \( V \) and \( S \) are the volume and surface of the body in the reference configuration, respectively, \( S_t \) is the bonded surface cohesive region, and \( T^i \) are contravariant components of the nominal surface tractions. An incremental version of the PVW (16) is used for the numerical solution. The displacement fields are approximated in terms of planar eight-noded isoparametric elements. The volume integral in Eq. (16) is carried out by using \( 2 \times 2 \) integration points within each element.

The region analyzed numerically is specified by \(-a \leq x_2 \leq a \) and \( 0 \leq x_1 \leq h \) in terms of the reference coordinates \( x_i \) (see Fig. 3(b)). The plunger is pressed against the surface \(-a \leq x_2 \leq -d \) at \( x_1 = 0 \), with the plunger displacement \( A \) in the \( x_1 \) direction. The plate is supported on the back surface \( d \leq x_2 \leq a \) at \( x_1 = h \) with zero displacement in the \( x_1 \) direction.
The cohesive plane, on which shear localization and failure is modeled, coincides with the $x_1$ axis. In all but two of the simulations, the gap $d$ between the plunger and the cohesive plane is taken to be zero. Full sticking is assumed at both the plunger and the support so that zero displacement in the $x_2$ direction is prescribed at these surfaces. Even though this is a plane strain analysis, the displacements in the $x_2$ direction are prescribed to be zero at the lower edge $x_2 = -a$, for $0 \leq x_1 \leq h$, thus modeling an effect similar to that at the centerline for circular geometries such as that in Fig. 1. The surface at $x_2 = a$ is traction-free. The initial geometry is here taken to be specified by $h/a = 0.8$. Figure 4(a) shows the initial mesh in the vicinity of the cohesive interface, with $64 \times 4$ uniform quadrilaterals on each side of the interface. The length of one square element inside this uniformly meshed region is denoted $D_0$.

To adequately resolve the fracture process, the length of the active process region (i.e., the region where $\lambda > \lambda_1$) should not be smaller than three or four times this element length $D_0$. Figure 4(b) shows the deformed mesh at $\Delta/h = 0.136$ for the reference case (case 1). This is just before the point where the full cross section at the interface has attained $\lambda = \lambda_1$, i.e., the equivalent of localization in the cohesive plane. A numerical difficulty encountered in this case with $d = 0$ is that nodal points on the plunger side moved past the plunger corner resulting in significant mesh distortion. This difficulty was resolved by prescribing that no nodal points on the plunger side of the interface can move past the current location of the corner of the plunger. The problem does not arise in analyses where the value of the normalized peak stress $\tilde{\tau}/\tau_Y$ is sufficiently small, but becomes pronounced when the value of the normalized peak stress is in the upper range considered in this paper.

4 Trends in Cropping Force and Work

In this section the results from a selection of simulations will be presented to expose the roles of the parameters identified in Sec. 2 on the cropping force–displacement behavior and the macroscopic work of cropping. A total of 21 simulations have been carried out. The dimensionless parameters for each simulation are listed in Table 1 and identified by a case number. The four primary parameters are $h/R_5$, $\tilde{\tau}/\tau_Y$, $N$, and $\tau_Y/E$. The thickness of the plate enters only through $h/R_5$. An alternative dimensionless parameter to $h/R_5$ is

$$h\tau_Y = \frac{E}{\Gamma_0 \tau_Y h}$$

which is also listed in Table 1.

Table 1 Simulation cases*

<table>
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<tr>
<th>Case</th>
<th>$h \Gamma_0^{-1}$</th>
<th>$\tilde{\tau}/\tau_Y$</th>
<th>$N$</th>
<th>$\tau_Y/E$</th>
<th>$d/R_5$</th>
<th>$\sigma_N/\sigma_Y$</th>
<th>$h\tau_Y$</th>
<th>$\Gamma_{CROP}/\Gamma_0$</th>
<th>$\tau_{YCROP}/\tau_Y$</th>
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*In all calculations: $\nu = 0.3$, $\lambda_1 = 0.15$, $\lambda_2 = 0.5$, and $\delta^0/\delta^* = 1$ (except for 21, $\delta^0/\delta^* = 1/2$). Case 1 is the reference case and denoted by a subscript $R$ in the text and figures.

$^b h \tau_Y / \Gamma_0$, given by (17), is not an independent parameter.
the predictions due to the dominance of shearing in the cropping process.

4.1 The Role of $h/R_s$—Specifically, the Role of the Specific Microscopic Work of Fracture $\Gamma_0$ and/or the Plate Thickness $h$. In this set of simulations, $N$ and $\tau_f/E$ are fixed and $h/R_s$ is varied for two choices of $\varepsilon/\tau_f$. By (17), for each choice of $\varepsilon/\tau_f$, varying $h/R_s$ is equivalent to varying $h\tau_f/\Gamma_0$. In other words, if one considers all the other dimensional parameters as being fixed, this set of simulations is equivalent to varying only $\Gamma_0$ with $h$ fixed or varying $h$ with $\Gamma_0$ fixed. For this set of simulations, varying only $\Gamma_0$ is accomplished by varying only $\delta^r$ [cf. (13)]. Figure 5 displays the dimensionless force–displacement curves. The influence of the parameter $h/R_s$ (or, equivalently, $h\tau_f/\Gamma_0$) on the force-displacement behavior is significant, especially so considering that dimensionless results for any model based only on plastic shearing alone cannot contain a dependence on $h$, at least assuming that there is no gap $d = 0$. This assertion follows from the dimensional argument that the only parameter containing a length dimension available to combine with $h$ to form a dimensionless parameter is $\Gamma_0$, e.g., $h/R_s$ or $h\tau_f/\Gamma_0$.

Values of the two dimensionless measures of the specific cropping work are given in Table 1. While it is essential to make use of dimensionless parameter combinations for problems such as cropping, which have a large number of independent parameters, relations among dimensionless variables often mask connections between dimensional quantities. This is particularly true for the specific work of cropping. Thus, to clearly display the effect of the specific microscopic work of fracture $\Gamma_0$ on the specific work of cropping $\Gamma_{CROP}$, normalized variables are used in Fig. 6 with $\Gamma_{CROP}/(\Gamma_0 h)$ plotted as a function of $\Gamma_0/\Gamma_0 h$ for plates of the same thickness $h$. Here, and subsequently, the subscript $R$ denotes a value from Table 1 for the reference case 1. These same curves provide the relation between $\Gamma_{CROP}/(\Gamma_0 h)$ and $(h\tau_f)/h$ for plates having the same $\Gamma_0$. Clearly the specific microscopic work of fracture has significant influence on the specific macroscopic cropping work, even though $\Gamma_0$ is a small fraction of $\Gamma_{CROP}$ (cf. Table 1). The quantitative predictions in Fig. 6 back up the earlier modeling of Atkins [1-4], who seems to be the first to view cropping as a shear fracture problem and not just a plastic shear-off process.

The curves in Fig. 6 also reveal the effect of plate thickness on the specific cropping work for the reference material. Doubling the thickness decreases $\Gamma_{CROP}$ by about 30%, and vice versa. Recall that both $\Gamma_{CROP}$ and $\Gamma_0$ are defined as energy per separated unit area, with units Jm$^{-2}$. Thus, the simulations predict that the total work required to crop a plate of the considered material increases by only a factor of about 1.4 when the thickness of the plate is doubled—much less than the factor of 2 expected if $\Gamma_{CROP}$ were independent of thickness. Conversely, cropping a plate of this material with half the thickness only reduces the total cropping work by a factor of about 0.7 rather than 0.5. This dependency on plate thickness is not at all obvious. However, it follows directly from dimensional arguments that there must be a dependence of $\Gamma_{CROP}$ on $h$ if there is a dependence on $\Gamma_0$—as noted above, the only length parameter in the problem that can be used to form a dimensionless parameter involving $\Gamma_0$ is $h$.

The several stages of the cropping process are indicated in Fig. 7 for the reference case. The entire cropping region undergoes extensive plastic yielding through the thickness of the plate prior to the onset of any crack growth. The fully plastic nature of the cropping process, emphasized in the Introduction, is evident in Fig. 7 where large nonlinear displacements due to plasticity occur prior to any cracking. At the maximum load, the shear cracks emerging from the corners of the platens have grown to a length of approximately $h/20$. With further imposition of the cropping displacement, each zone of shear decohesion extends towards the center of the plane until, at $\Delta/h \approx 0.14$, they merge at the center. Prior to this point in the process, significant plastic deformation takes place outside the cohesive zone. However, for $\Delta/h > 0.14$, the deformation is mainly confined to the cohesive zone, the cracks grow toward each other, and the cropping force drops dramatically until the cracks connect at the center at $\Delta/h \approx 0.16$.

There are some differences between the sequence of events occurring in Fig. 7 and those laid out in the early work of Johnson and Slater [14]. These authors argued that at the time their paper was written there was no evidence for shear cracking prior to the maximum punch force, and they suggest that the maximum is a result of shear localization. In their view, cracking first occurs after the maximum load is attained. Specific details such as these will depend on the tendency for a given material to undergo shear localization prior to shear damage. To some extent, the sequence of events can be explored within the context of the present model by varying the cohesive zone parameters. For example, decreasing the shear strength $\tilde{\tau}$ while fixing, or increasing, the specific work

\[ \frac{\Gamma_{CROP}}{(\Gamma_0 h)} \approx \frac{\Gamma_0}{M h} \]
force vanishes when the cracks connect. Taken into account in the present model and, thus, the cropping expected to play once a shear crack has developed. Friction is not in this set of simulations values and ping work from Table 1 for these three cases are .

4.3 The Role of Strain Hardening $N$. The simulated force–displacement curves in Fig. 9 show the effect of varying $N$ with the other dimensionless parameters held fixed. Increased strain hardening elevates the cropping force, but makes it easier to attain the tractions required to cause cohesive shear. The result is that higher strain hardening decreases the normalized displacement $\Delta/h$ at which the cropping force become zero. With fixed $\tau_Y$, the cropping work is also a strong function of $N$, varying from $\Gamma_{CROP}/\tau_Y = 0.0973$ for $N = 0.33$ to $\Gamma_{CROP}/\tau_Y = 0.626$ for $N = 0.125$ (Table 1). Of course, for a family of alloys, processing treatments that increase $N$ are usually accompanied by a decrease in $\tau_Y$, and thus, when that is the case, a change in the parameter $\tau_Y/E$ must also be taken into account.

4.4 The Role of $\tau_Y/E$. In this subsection, the effect of changes in $\tau_Y/E$ with $h/R_S$, $\tilde{\tau}/\tau_Y$, and $N$ held fixed are examined. To carry out these calculations, $E$ was changed, but to ensure that $h/R_S$ did not change, $\Gamma_0$ was also changed such that the product $E\Gamma_0$ was fixed. The normalized force displacement curves are shown in Fig. 10, where the influence of $\tau_Y/E$ is seen to be significant. The role of $\tau_Y$ (or $E$) is not easy to decipher because it is present in three (two) of the dimensionless parameter combinations. While the dimensionless specific work of cropping $\Gamma_{CROP}/\tau_Y h$ is strongly affected by variations in $\tau_Y/E$, the second measure in Table 1 $\Gamma_{CROP}/\Gamma_0$ is only weakly dependent on $\tau_Y/E$. A fourfold change in $\tau_Y/E$ (with $h/R_S$, $\tilde{\tau}/\tau_Y$, and $N$ fixed) produces only about a 25% change in $\Gamma_{CROP}/\Gamma_0$. The important conclusion to be drawn is that the cropping work as measured by $\Gamma_{CROP}/\Gamma_0$ depends primarily on $h/R_S$, $\tilde{\tau}/\tau_Y$, and $N$. of fracture $\Gamma_0$ promotes early localization relative to shear cracking. In addition, Johnson and Slater [4] discuss the role friction is expected to play once a shear crack has developed. Friction is not taken into account in the present model and; thus, the cropping force vanishes when the cracks connect.

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4.5 The Role of Prestress $\sigma_{PS}/\sigma_Y$. To illustrate one possible application of the model, the effect of uniform prestraining [1] is considered. Assume the plate is strained in uniaxial tension beyond yield to a stress $\sigma_{PS}$ then released to the unstressed state prior to cropping. This prestressing is accounted for in the cropping simulations by simply expanding the starting radius of Mises yield surface from its initial value associated with $\sigma_Y$ to the value associated with $\sigma_{PS}$. Case 17, with $\sigma_{PS}/\sigma_Y = 1.2$, and case 18, with $\sigma_{PS}/\sigma_Y = 2$, given in Table 1, have their other parameters identical to those of the reference, case 1. (In Table 1, for all the other cases, $\sigma_{PS}/\sigma_Y = 1$ has been used to indicate there has been no plasticity due to prestress.) The cropping force–displacement curves in Fig. 11 reveal that a small prestress $\sigma_{PS}/\sigma_Y = 1.2$ has essentially no effect on cropping. However, the larger value $\sigma_{PS}/\sigma_Y = 2$ has a noticeable effect on the cropping curve and reduces the specific work of cropping by about 10%.

4.6 The Role of the Gap $d/h$. A few computations have been carried out to study the effect of a cropping gap. Specifically, for case 19, the initial gap between the edge of the plunger and the cohesive plane was taken to be two element sizes, $d = 2D_0$, and the same gap is assumed at the edge of the supporting tool. Thus, the cohesive plane coincides with the center of the gap. If the mesh in Fig. 4(a) is used, strong mesh distortion at the edge of the plunger leads to early breakdown of the computation. Therefore, a specially refined mesh is used around the point where intense straining is induced at the edge of the plunger, as shown in Fig. 12(a). This enables the computation to proceed, but the material in the gap rotates significantly, even to the extent that it overlaps the material under the plunger. Consequently, an extra condition is introduced stipulating that the material in the gap cannot penetrate the plunger, as illustrated by the deformed mesh in Fig. 12(b). This contact condition is taken to be frictionless sliding.

The normalized force–displacement curves for two gaps sizes $d/h = 0.031$ and 0.047 are presented in Fig. 13 along with that for the reference case (case 1) which has no gap but otherwise identical parameters. A gap has a significant influence resulting in larger plastic deformation accompanying the cropping process. For the smaller of the two gap sizes, $d/h = 0.031$, the cropping work is approximately twice that of the reference case (cf. Table 1).

As indicated by the above discussion, the large plastic strain and extensive rotation accompanying the shear decohesion process is challenging to simulate, especially when a gap is present. More extensive studies accounting for the gap are clearly required. Handbook rules exist recommending maximum gap to thickness ratios for cropping; modeling these rules is part of the challenge ahead. It should also be mentioned that the effect of changing the relative tensile strength in the cohesive zone

![Fig. 10](http://appliedmechanics.asmedigitalcollection.asme.org/ on 08/31/2016 Terms of Use: http://www.asme.org/about-asme/terms-of-use)

![Fig. 11](http://appliedmechanics.asmedigitalcollection.asme.org/ on 08/31/2016 Terms of Use: http://www.asme.org/about-asme/terms-of-use)

![Fig. 12](http://appliedmechanics.asmedigitalcollection.asme.org/ on 08/31/2016 Terms of Use: http://www.asme.org/about-asme/terms-of-use)
The role of a gap between the tool corners and the cohesive plane $d/h$ on the normalized force–displacement curve with the other dimensionless parameters held fixed and equal to those of the reference case. Cases: 1, 19, and 20.

$\Delta / h = 0.0427$

$\frac{d}{h} = 0.031, 0.047$

$\frac{\tau_n}{\tau_f} = 3.12$

$N = 0.185$

$\frac{\tau_n}{E} = 0.000785$

$\sigma = (\delta^2 / \delta \epsilon^2)\bar{\epsilon}$ has not been studied in the presence of a gap when it is more likely to be important [5].

5 Concluding Remarks

Cropping falls within the larger field concerned with the mechanics and physics of cutting [18]. The present paper has introduced a model which links macroscopic aspects of cropping, such as the cropping force–displacement behavior and the work of cropping, to properties at the microscopic scale such as the fundamental fracture energy of the material and its shear strength. Predictions based on the model have been presented to reveal trends in this relationship. In the examples presented in the body of the paper, the specific macroscopic work of cropping ranges from 2 to 13 times the specific microscopic work of fracture. Nevertheless, even when the microscopic work of fracture comprises only a small fraction of the total work, it has been shown to play a critical role in establishing the macroscopic behavior. The present cropping study parallels earlier efforts to predict the macroscopic mode I fracture toughness of ductile metal alloys in terms of more fundamental microscopic material properties [6]. In so doing, the present study advances the case made by Atkins [1–4] that cropping should be viewed as a fracture problem. In the terminology of nonlinear fracture mechanics, cropping is a large scale yielding fracture problem. Cropping presents its own special challenges owing to the large plastic strains that inevitably accompany the process and the fact that shear localization and shear fracture are less well understood in terms of fundamental material mechanisms than the tensile fracture of ductile metals under high stress triaxiality. The challenges are both computational and physical.

As mentioned in connection with the introduction of the reference case, the choices of the peak shear strength and work of fracture characterizing the cohesive zone have not been directly calibrated with experiments. For example, it is possible that the microscopic parameters of the steel DH36 lie outside the range covered by the simulations in this paper. The fact that the plunger displacement at failure $\Delta / h = 0.5$ for the DH36 test in Fig. 1(b) is larger than any of the values found in the present simulations suggests this might be the case. The main purpose of the present paper has been to expose trends in the roles of the parameters controlling cropping. Future efforts will be undertaken to make direct comparisons with cropping experiments. Finally, it must be reemphasized that important factors could be added to the present model which have not been taken into account, such as tool deformation, friction, and material temperature and rate dependence.

References