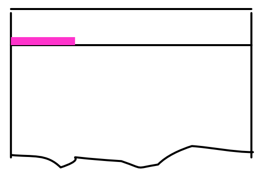
SLIDES ON DELAMINATION MECHANICS

with applications to films, coatings & multilayers

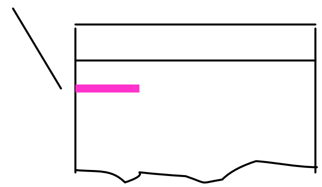
Delamination modes

Mixed mode edge crack on interface



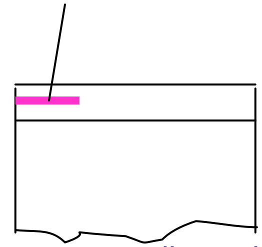
Tension & compression in film

Mode I substrate crack



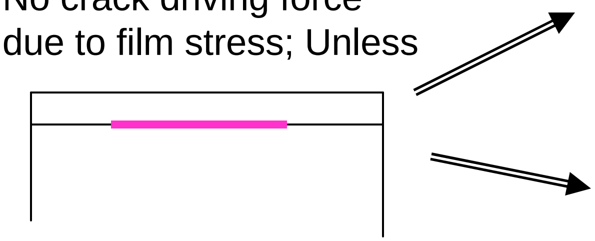
Tension in film

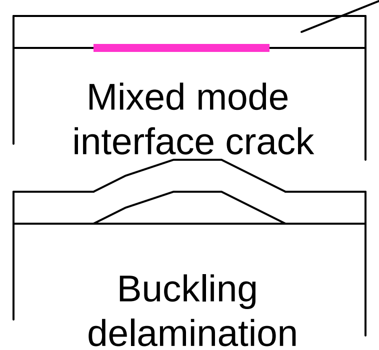
Mode I crack in film



Stress gradient with tension in film

No crack driving force due to film stress; Unless



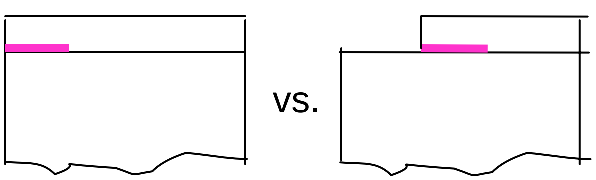


Thermal gradient with interruption of heat transfer across crack.

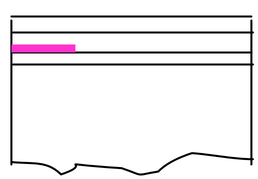
Compression in film

Applications to thermal loadings

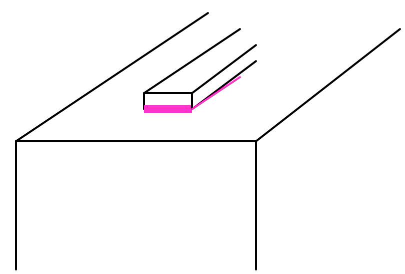
Other issues:



Edge effects



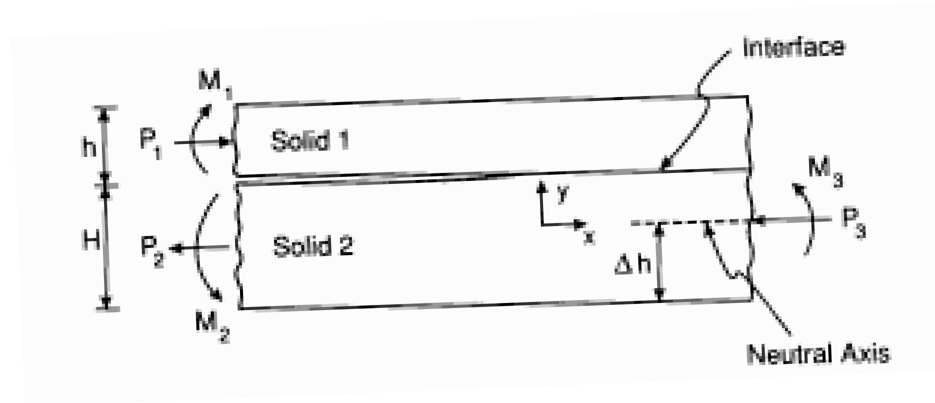
Multi-layers



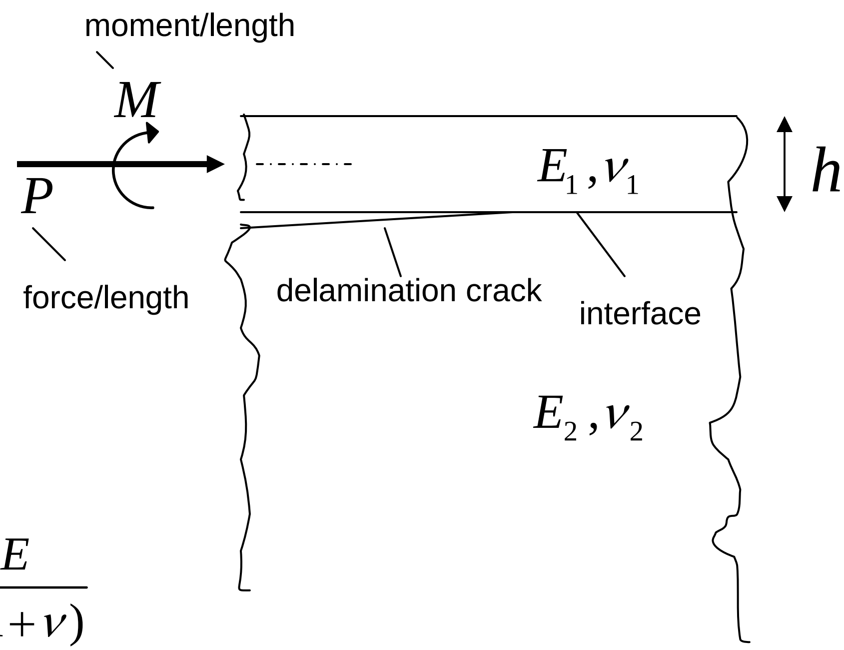
3D effects for film strips

BASIC ELASTICITY SOLUTION FOR INFINITE ELASTIC BILAYER WITH SEMI-INFINITE CRACK

Equilibrated loads. General solution for energy release rate and stress intensity factors available in Suo and Hutchinson (1990)



Infinitely thick substrate--
Primary case of interest for thin films and coatings on thick substrates



Dundurs' mismatch parameters for plane strain:

$$\alpha_D = \frac{\bar{E}_1 - \bar{E}_2}{\bar{E}_1 + \bar{E}_2}, \quad \bar{E} = \frac{E}{(1-\nu^2)}$$

$$\beta_D = \frac{1}{2}\frac{\mu_1(1-2\nu_2) - \mu_2(1-2\nu_1)}{\mu_1(1-\nu_2) + \mu_2(1-\nu_1)}, \quad \mu = \frac{E}{2(1+\nu)}$$

For homogeneous case: $\alpha_D = \beta_D = 0$ If both materials incompressible: $\beta_D = 0$

α_D is the more important of the two parameters for most bilayer crack problems

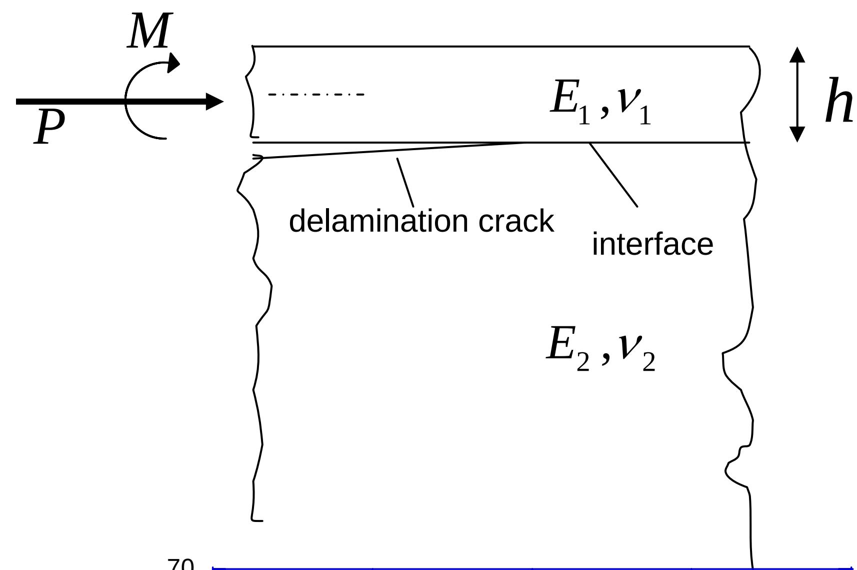
Take $\beta_D = 0$ if you can. It makes life easier!

Basic solution continued:

Energy release rate

$$G = \frac{1}{2\bar{E}_1}\left(\frac{P^2}{d} + 12\frac{M^2}{d^3}\right)$$

$$\bar{E} = E/(1-\nu^2)$$

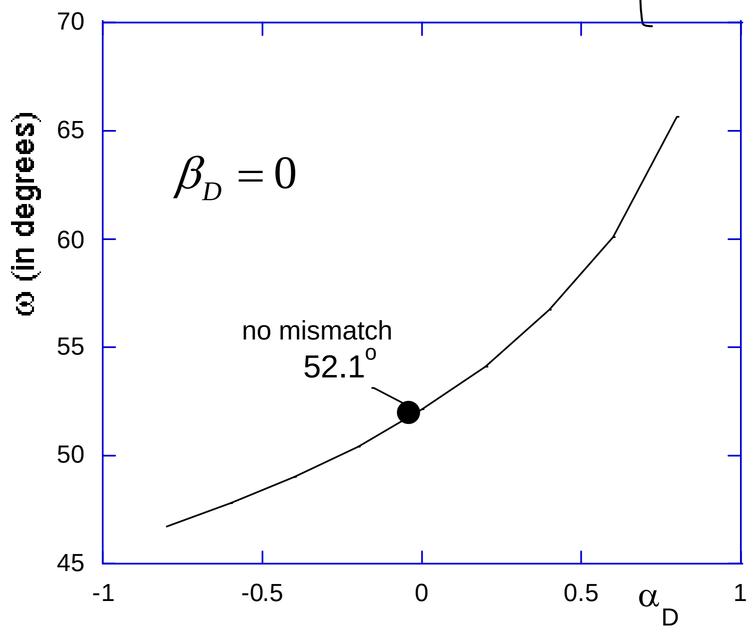


Stress intensity factors: $(\beta_D = 0)$
(see Hutchinson & Suo (1992) if second Dundurs' parameter cannot be taken to be zero)

$$K_I = \frac{1}{\sqrt{2}}\left[Pd^{-1/2}\cos\omega + 2\sqrt{3}Md^{-3/2}\sin\omega\right]$$

$$K_{II} = \frac{1}{\sqrt{2}}\left[Pd^{-1/2}\sin\omega - 2\sqrt{3}Md^{-3/2}\cos\omega\right]$$

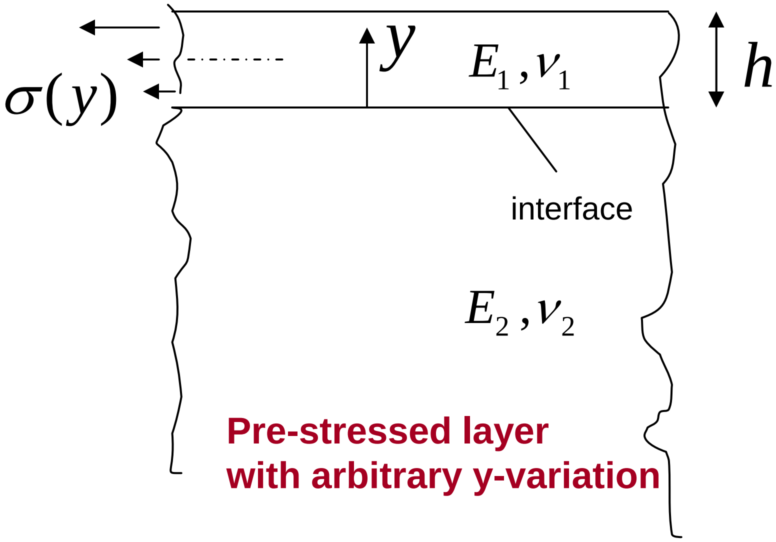
where $\omega(\alpha_D)$ is shown as a plot and is tabulated in Suo & Hutch.



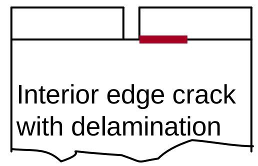
Note: For any interface crack between two isotropic materials,

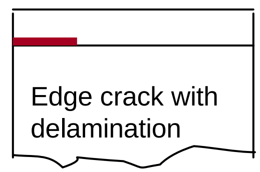
$$G = \frac{1-\beta_D^2}{2}\left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right)\left(K_I^2 + K_{II}^2\right)$$

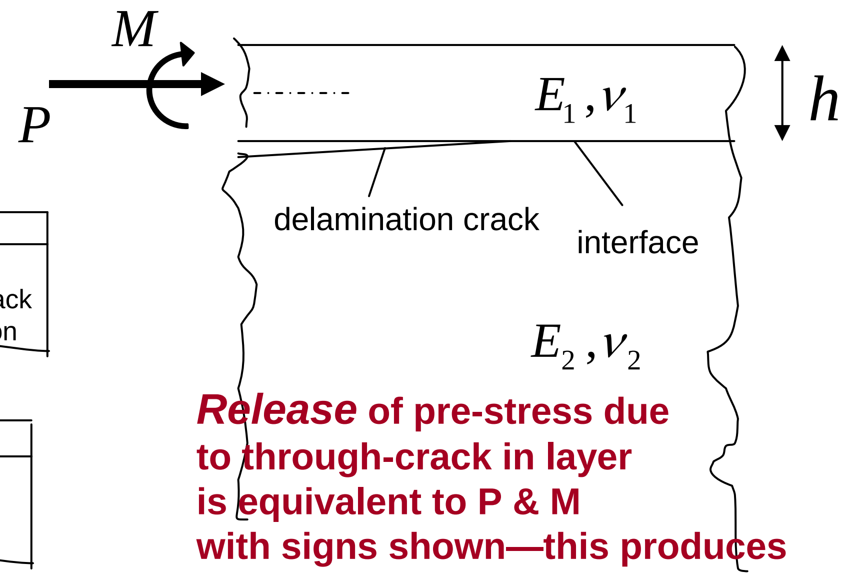
Application to films & coatings



Pre-stressed layer
with arbitrary y-variation







Release **of pre-stress due**
to through-crack in layer
is equivalent to P & M
with signs shown—this produces
the stress intensities at tip

$$P = \int_0^h \sigma(y)dy$$

$$M = \int_0^h \sigma(y)(y - \tfrac{1}{2}h)dy$$

Pre-stress can arise from thermal expansion mismatch, deposition processes, mechanical loading (bending or stretching of film/substrate), drying or absorption of moisture, etc.

Simplest example: uniformly stressed film on an interface with no mismatch

$$P = \sigma h, \;\; M = 0, \;\; \omega = 52.1^o,$$

$$G = \sigma^2 h / 2\bar{E}$$

$$\sigma > 0: \quad K_I = 0.434\sigma\sqrt{h}, \quad K_{II} = 0.556\sigma\sqrt{h}$$

$$\sigma < 0: \quad K_I = 0, \quad K_{II} = -0.707\sigma\sqrt{h}$$

Closed, mode II crack. Only valid if friction is neglected.

Application to films & coatings, continued: Role of elastic mismatch

Illustrative example: uniformly stressed film: tension

$$P = \sigma h,\ \ M = 0,\ \ \sigma > 0: \quad K_I = \sigma\sqrt{h}\cos\omega/\sqrt{2},\ \ K_{II} = \sigma\sqrt{h}\sin\omega/\sqrt{2}$$

Energy release rate: $G = \sigma^2 h / 2\bar{E}$, Measure of mode mix: $\psi = \tan^{-1}(K_{II}/K_I)$

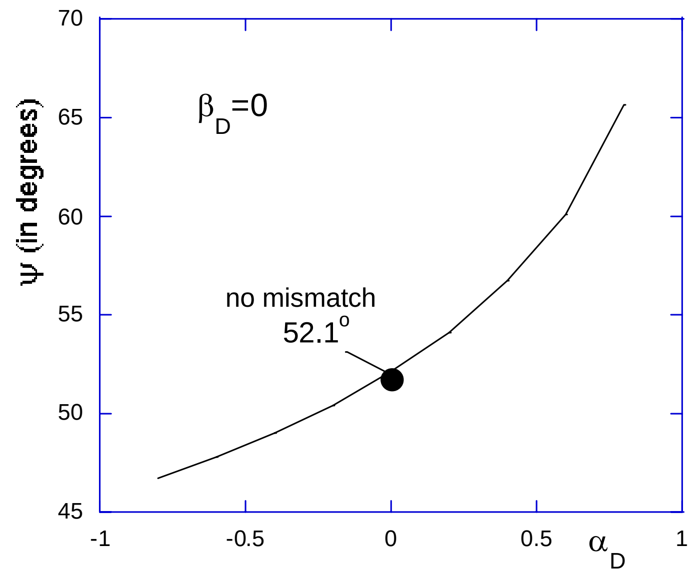
For this problem: $\psi = \omega(\alpha_D)$

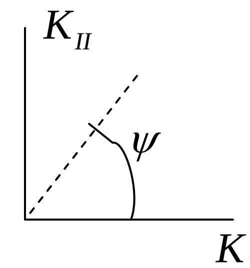
$$\alpha_D = \frac{\bar{E}_1 - \bar{E}_2}{\bar{E}_1 + \bar{E}_2}$$

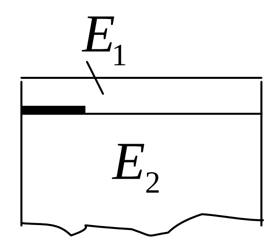
$\bar{E}_1 = \bar{E}_2 \quad \Rightarrow \alpha_D = 0,\ \omega = 52.1^o$

$\bar{E}_1 = 3\bar{E}_2 \quad \Rightarrow \alpha_D = 1/2,\ \omega = 57^o$

$\bar{E}_1 = \bar{E}_2/3 \Rightarrow \alpha_D = -1/2,\ \omega = 48^o$

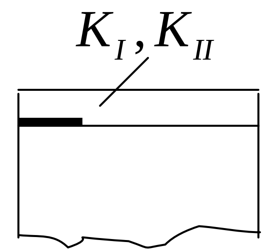






For modest mismatches (metals on metals or ceramics), the role of the elastic mismatch on the mode mix is relatively minor. However, for large mismatches (e.g., metals or ceramics on polymers or elastomers, or vise versa), the influence on the mode mix can be large.
But note that the mismatch does not effect the energy release rate for long cracks (steady-state).

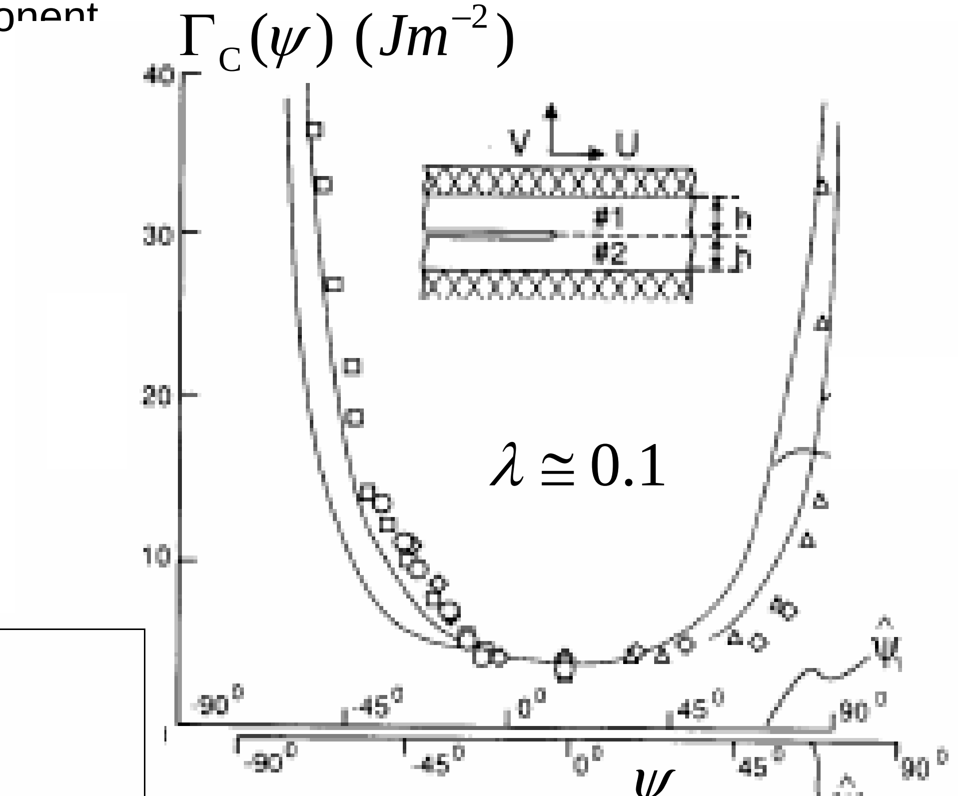
Interface toughness—the role of mode mix



Experimental finding: The energy release rate required to propagate a crack along an interface generally depends on the mode mix, often with larger toughness the larger the mode II component.

Interface Toughness: $\Gamma_C(\psi)$

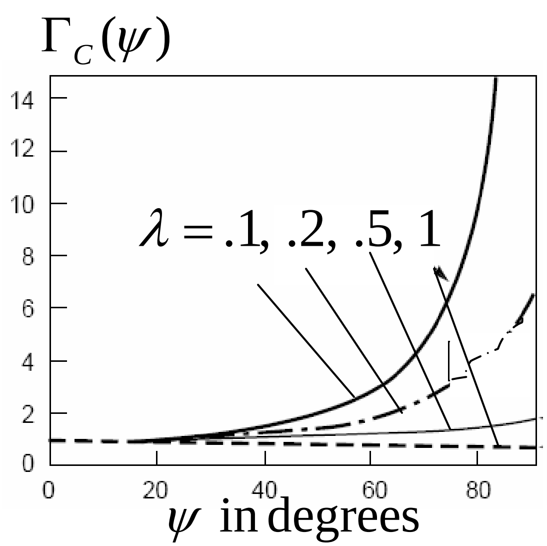
Propagation condition: $G = \Gamma_C(\psi)$

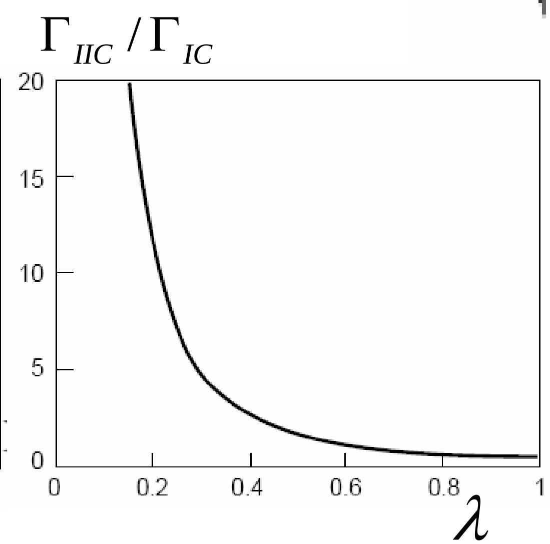


Liechti & Chai (1992) data for an epoxy/glass interface.

A phenomenological interface toughness law

$$\Gamma_C(\psi) = \Gamma_{IC}\left(1 + \tan^2((1-\lambda)\psi)\right)$$



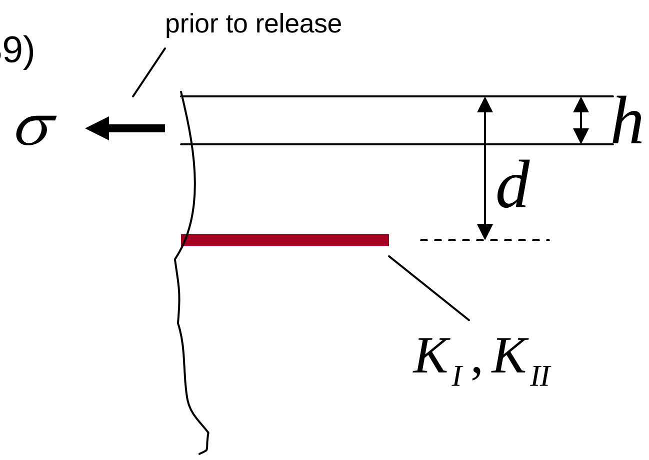


$\lambda = 1 \Rightarrow$ no mode dependence

$\lambda << 1 \Rightarrow$ significant mode dependence

Mode I cracking in substrate driven by tensile stresses in film or coating

The solution to the problem depicted is given in Suo & Hutch (1989) for general elastic mismatch between the top layer and the substrate. See Drory, Thouless & Evans (1988) for experimental observations for metal/glass systems.



Neglecting elastic mismatch the basic solution gives

$$K_I = \frac{\sigma h}{\sqrt{2d}}\left(\cos\omega + \sqrt{3}\left((d-h)/d\right)\sin\omega\right)$$

$$K_{II} = \frac{\sigma h}{\sqrt{2d}}\left(\sin\omega - \sqrt{3}\left((d-h)/d\right)\cos\omega\right)$$

Depth of mode I crack

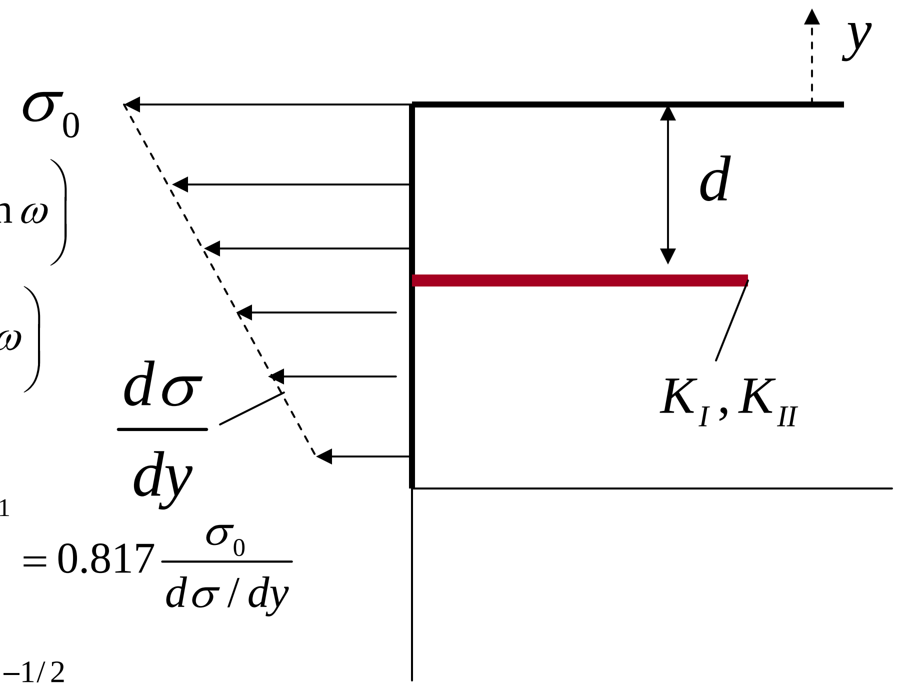
$$K_{II} = 0 \text{ with } \omega = 52.1^o \Rightarrow \frac{d}{h} = 3.86, \quad K_I = 0.586\sigma\sqrt{\mathrm{h}} \text{ and } \quad G = 0.343\sigma^2 h/\bar{E}$$

Compare with mixed mode delamination along interface

$$K_I = 0.434\sigma\sqrt{\mathrm{h}}, \; K_I = 0.556\sigma\sqrt{\mathrm{h}} \quad \text{and} \quad G = 0.50\sigma^2 h/\bar{E}$$

Substrate delamination as mode I crack propagation is observed in systems where the interface is relatively tough and the substrate is brittle. The stress in the film or coating must be in tension. No mode I path exists in the substrate if the stress is compression.

Mode I cracking within a film or coating driven by stress gradients



Basic solution gives:

$$K_I = \frac{1}{\sqrt{2}}\sigma_0 d^{1/2}\cos\omega + \frac{1}{\sqrt{2}}\frac{d\sigma}{dy}d^{3/2}\left(-\cos\omega + \frac{1}{2\sqrt{3}}\sin\omega\right)$$

$$K_{II} = \frac{1}{\sqrt{2}}\sigma_0 d^{1/2}\sin\omega + \frac{1}{\sqrt{2}}\frac{d\sigma}{dy}d^{3/2}\left(\sin\omega + \frac{1}{2\sqrt{3}}\cos\omega\right)$$

Mode I crack:

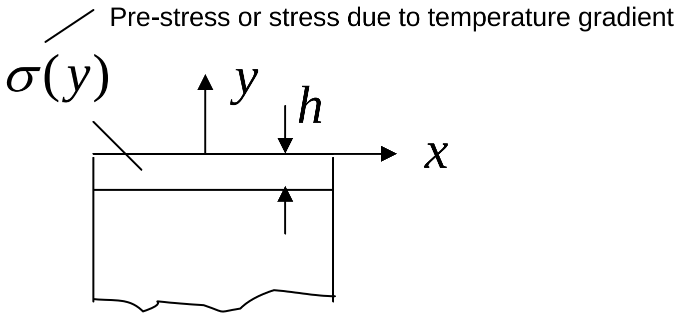
$$K_{II} = 0 \;\; (\omega = 52.1^o) \;\Rightarrow\; d = \frac{\sigma_0}{d\sigma/dy}\left(1 + \frac{\cot\omega}{2\sqrt{3}}\right)^{-1} = 0.817\frac{\sigma_0}{d\sigma/dy}$$

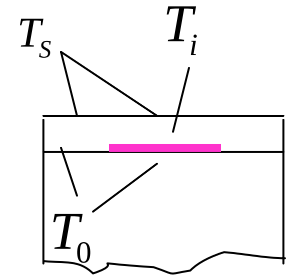
$$K_I = 0.657\sigma_0 d^{1/2} = 0.594\sigma_0^{\;3/2}(d\sigma/dy)^{-1/2}$$

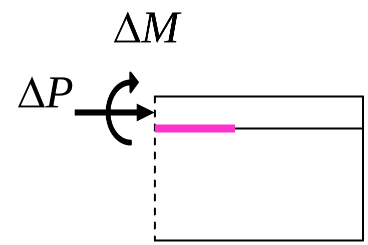
For a mode I crack to exist within layer with linear stress variation:

$$\sigma_0 > 0 \quad \& \qquad d\sigma/dy > 0.817\sigma_0/H \quad \Longrightarrow \quad d = 0.817\frac{\sigma_0}{d\sigma/dy} \;\;\&\;\; G = 0.353\frac{\sigma_0^{\;3}}{\bar{E}\,d\sigma/dy}$$

Delamination of an Interior Interface Crack in a Thermal Gradient







Basic loading involving the difference between the cracked & uncracked configurations

Fundamental observation: Given any in-plane stress variation dependent only on y, there are no stresses acting on interface in the interior of a film or multlayer.

--An interior interface crack has zero stress intensity

What can produce x-dependence and stress intensity?

--- proximity to a free edge or through-crack
--- buckling of film due to compressive stress
--- thermal gradient and low conductivity across crack

<u>Isolated interface crack in a thermal gradient</u>

Stress difference in central region of a long crack (cracked - uncracked):
The stress difference produces the crack tip intensities

$$\Delta\sigma = \frac{E_1}{1-\nu_1}\alpha_1 \Delta T\left(1-y/h\right), \quad \Delta T \equiv T_i - T_0 = \frac{T_S - T_0}{1+B}$$

Biot number for interface:

$$B = \frac{c_i h}{k_1}, \quad \text{heat flow} = q = -c_i\left(T_i - T_0\right)$$

Thermal conductivity in top layer

Interface thermal conductivity

Basic solution gives:

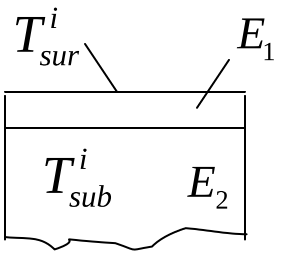
$$\Delta P = \frac{1}{2}\frac{E_1 h \alpha_1 \Delta T}{(1-\nu_1)}, \quad \Delta M = -\frac{1}{12}\frac{E_1 h^2 \alpha_1 \Delta T}{(1-\nu_1)}$$

$$G = \frac{1}{6}\left(\frac{1+\nu_1}{1-\nu_1}\right)E_1 h\left(\alpha_1\left(T_i - T_0\right)\right)^2$$

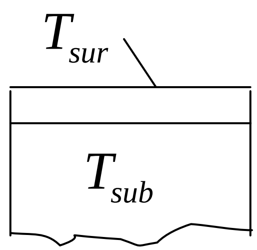
$$\psi = \tan^{-1}\left(\frac{\sqrt{3}\tan\omega + 1}{\sqrt{3} - \tan\omega}\right) = 82^o \quad (\omega = 52.1^o)$$

- G is small even with B=0 compared with result for edge crack.
- This is the long crack limit. It is an upper bound for shorter cracks.
- The crack tip experiences near mode II conditions

Thermal Barrier Coatings Subject to Temperature Gradient:
Interface Delamination on Cool-down

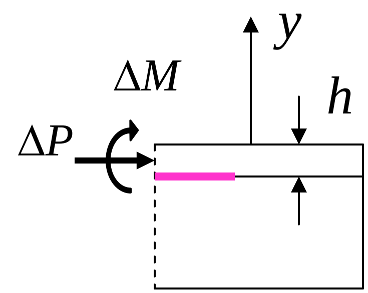


At highest operating temperature—stresses relax to zero



At any stage during cool-down—stress is thermo-elastic (no relaxation)

$$\Delta T_{sub} = T^{i}_{sub} - T_{sub}$$
$$\Delta T_{sur} = T^{i}_{sur} - T_{sur}$$
$$\Delta T_{sur/sub} = \Delta T_{sur} - \Delta T_{sub}$$
$$\Delta\sigma = \frac{E_1 \alpha_{tbc} \Delta T_{sur/sub}}{(1-\nu_1)}\left(1+\frac{y}{h}\right) + \frac{E_1 \Delta\alpha \Delta T_{sub}}{(1-\nu_1)}, \quad \Delta\alpha = \alpha_{sub} - \alpha_{tbc}$$



Basic solution gives:

$$G = \frac{(1+\nu_2)E_1 h}{6(1-\nu_2)}\left(\left(\alpha_{tbc}\Delta T_{sur/sub}\right)^2 - 3\alpha_{tbc}\Delta T_{sur/sub}\Delta\alpha\Delta T_{sub} + 3\left(\Delta\alpha_{tbc}\Delta T_{sub}\right)^2\right)$$

$$\tan\psi = \frac{\sqrt{3}\tan\omega\left(\alpha_{tbc}\Delta T_{sur/sub} - 2\Delta\alpha\Delta T_{sub}\right) - \alpha_{tbc}\Delta T_{sur/sub}}{\sqrt{3}\left(\alpha_{tbc}\Delta T_{sur/sub} - 2\Delta\alpha\Delta T_{sub}\right) + \tan\omega\,\alpha_{tbc}\Delta T_{sur/sub}}$$

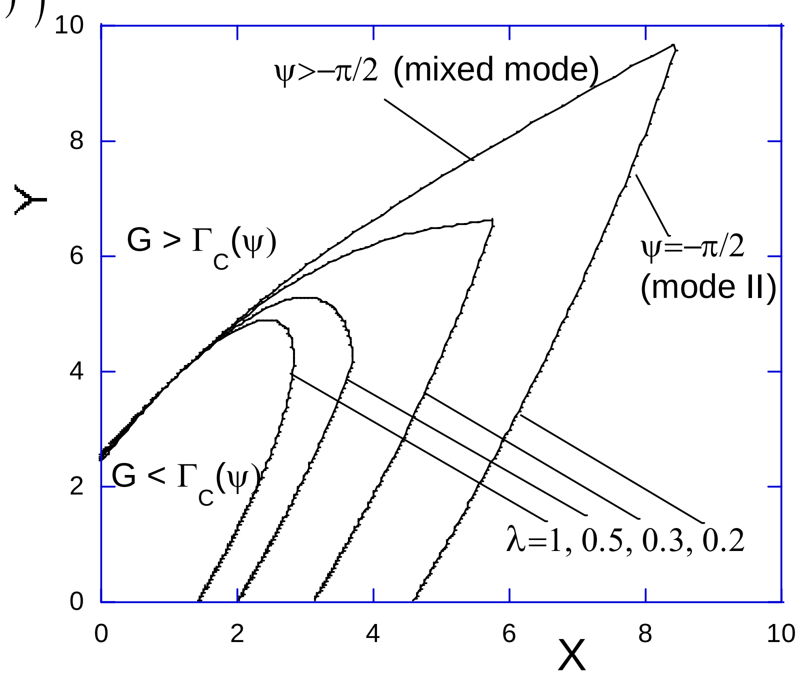
Interface toughness (see earlier slide):

$$\Gamma_C(\psi) = \Gamma_{IC}\left(1+\tan^2((1-\lambda)\psi)\right)$$

$$G = \Gamma_C(\psi) \Rightarrow Y^2 - 3YX + 3X^2 = 6\left(1+\tan^2\left((1-\lambda)\psi\right)\right)$$

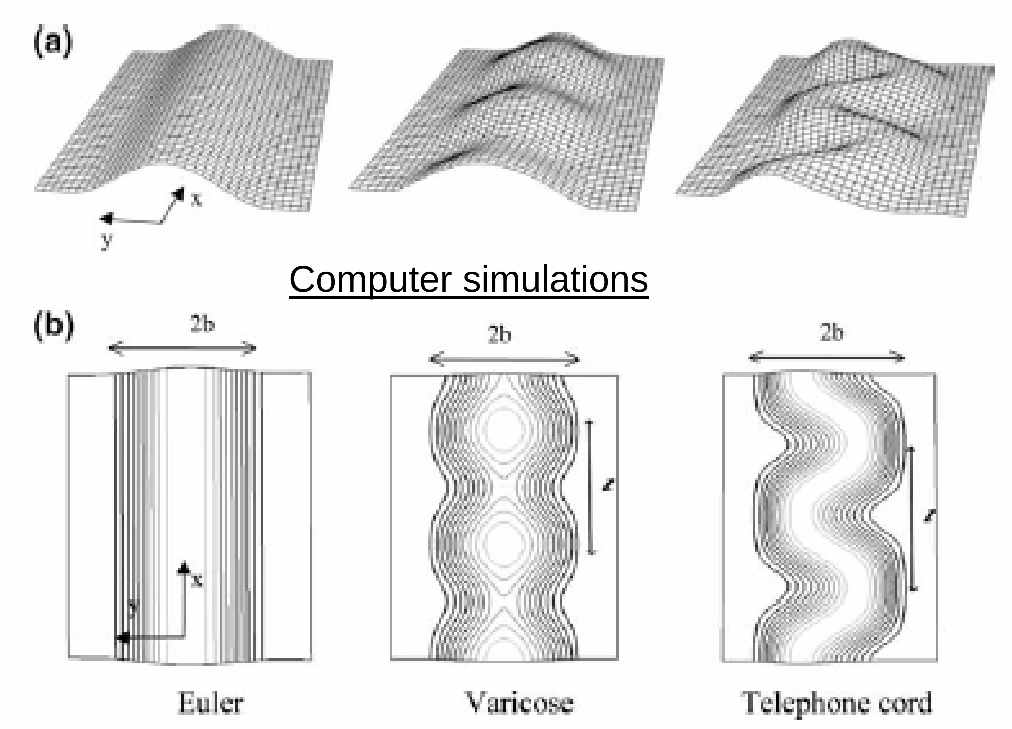
$$X = \frac{\Delta\alpha\Delta T_{sub}}{\sqrt{\frac{(1-\nu_1)}{(1+\nu_1)}\frac{\Gamma_{IC}}{E_1 H}}}, \qquad Y = \frac{\alpha_{tbc}\Delta T_{sur/sub}}{\sqrt{\frac{(1-\nu_1)}{(1+\nu_1)}\frac{\Gamma_{IC}}{E_1 H}}}$$

Delamination Contours for Cool-down $\omega = 54.1^o$

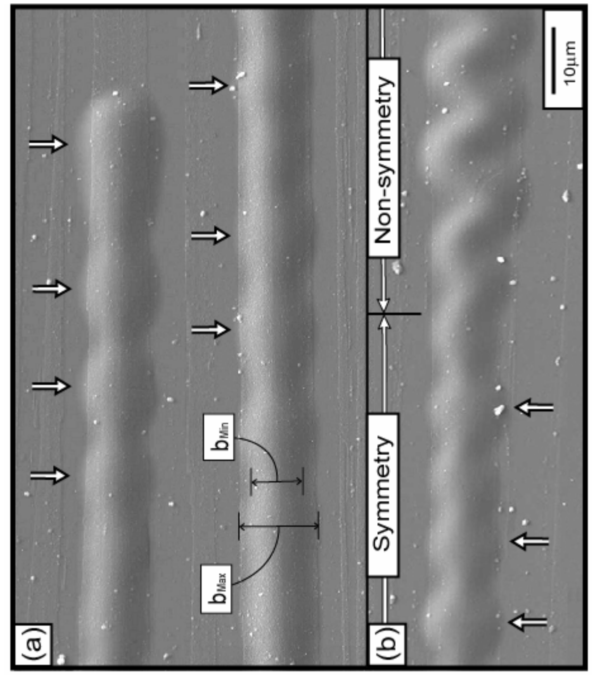


Buckle Delaminations: Interface cracking driven by buckling

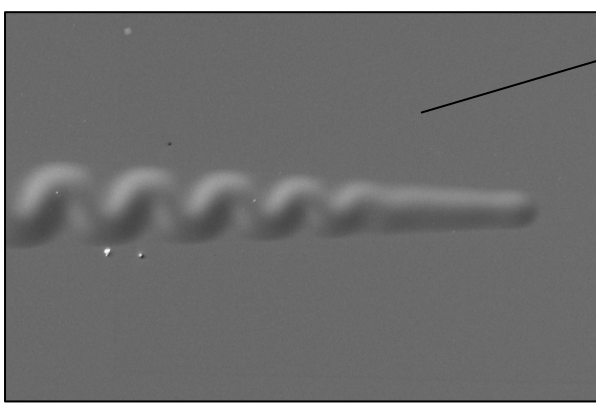
Three Morphologies: Straight-sided, Varicose and Telephone Cord



Straight-sided



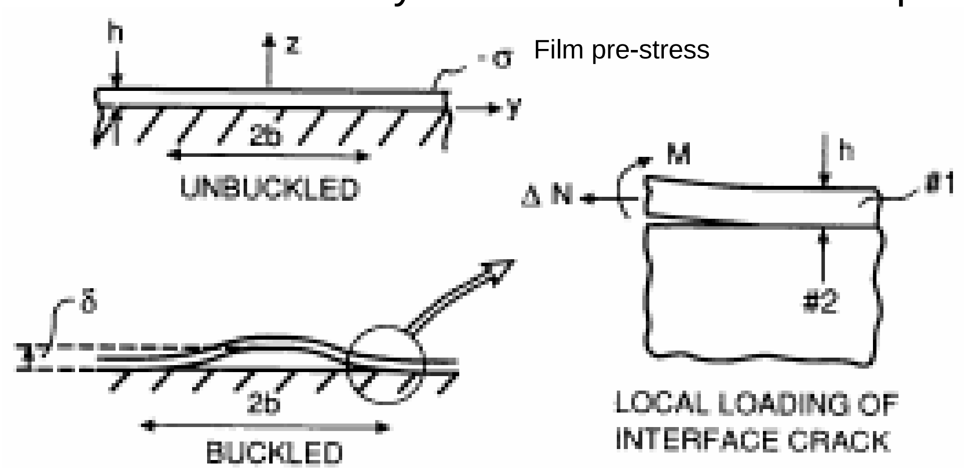
<u>Experimental observations</u>
200nm DLC film on silicon

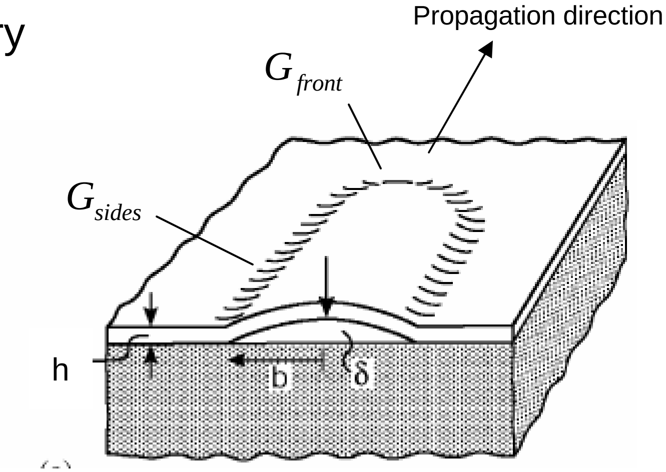


Propagation of a buckle delamination along a pre-patterned tapered region of low adhesion between film and substrate. In the wider regions the telephone cord morphology is observed. It transitions to the straight-sided morphology in the more narrow region and finally arrests when the energy release rate drops below the level needed to separate the interface.

Abbreviated Analysis of the Straight-Sided Buckle Delamination

A 1D analysis based on vonKarman plate theory





Buckle deflection:

$$w(y)=\frac{1}{2}\delta\left(1+\cos(\pi y/b)\right)$$

Average stress in buckled film:

$$\sigma_C=\frac{\pi^2}{12}\bar{E}_1\left(\frac{h}{b}\right)^2$$

In-plane compatibility condition

$$\frac{1}{\bar{E}_1}(\sigma-\sigma_C)=\frac{1}{2}\int_{-b}^{b}w'^2dy=\frac{\pi^2}{8b}\delta^2$$

Buckle amplitude:

$$\frac{\delta}{h}=\sqrt{\frac{4}{3}\left(\frac{\sigma}{\sigma_C}-1\right)}$$

At edge of buckle:

$$\Delta N=(\sigma-\sigma_C)h,\ \ M=\frac{\pi^2\delta}{2b^2}$$

Energy release rate and mode mix along sides from basic solution:

$$G_{sides}=\frac{h}{\bar{E}_1}(\sigma-\sigma_C)(\sigma+3\sigma_C)\qquad \tan\psi=\frac{4+\sqrt{3}(\delta/h)\tan\omega}{-4\tan\omega+\sqrt{3}(\delta/h)}$$

Energy-release rate can also be obtained from direct energy change calculation

Mode mix depends on the amplitude of The buckle

Energy release rate along propagating front

$$G_{front}=\frac{1}{2b}\int_{-b}^{b}G_{sides}dy=\frac{h}{\bar{E}_1}(\sigma-\sigma_C)^2$$

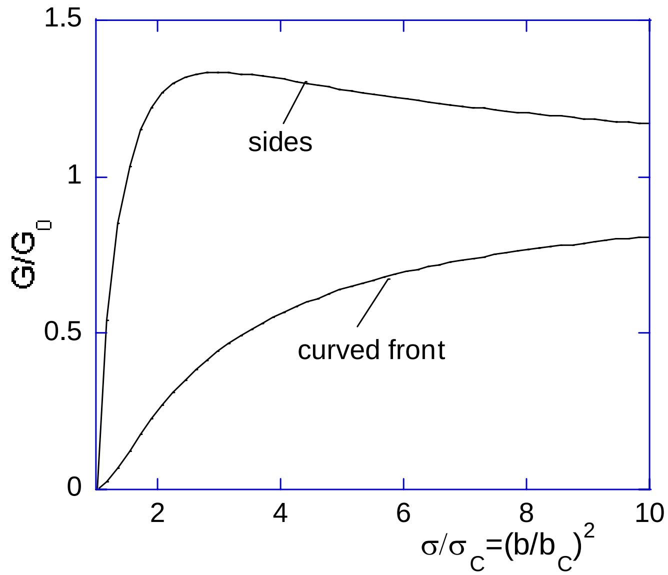
Plots are given on next overhead

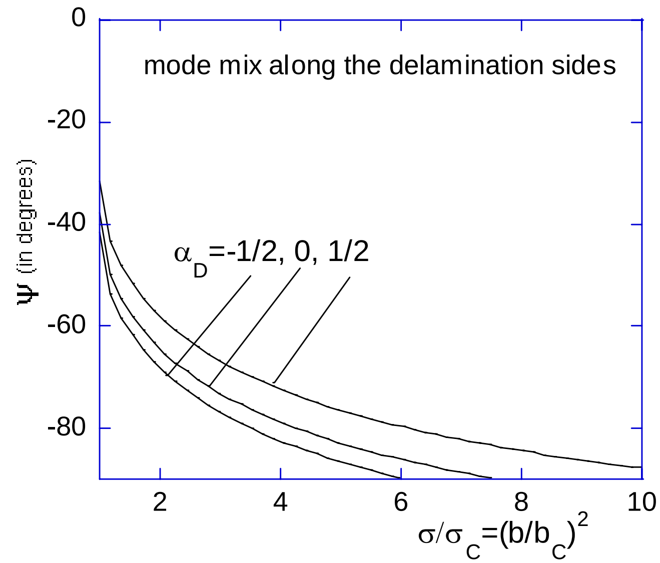
Energy release rate and mode mix on sides of Straight-sided buckle delamination

$$\sigma_C = \frac{\pi^2}{12}\bar{E}_1\left(\frac{h}{b}\right)^2$$ Stress at onset of buckling

$$\frac{b_C}{h} = \sqrt{\frac{\pi^2}{12}\frac{\bar{E}_1}{\sigma}}$$ Half-width at onset of buckling

$$G_0 = \frac{1}{2}\frac{\sigma^2 h}{\bar{E}_1}$$ Energy/area available for release in planes strain



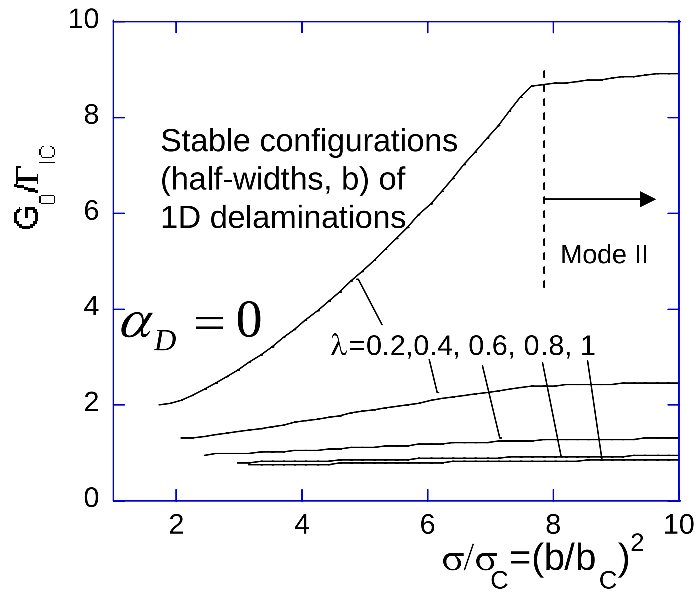


Half-width of straight-sided delamination

Impose: $G = \Gamma_{IC}\, f(\psi), \qquad f(\psi) = 1 + \tan^2((1-\lambda)\psi)$

See earlier slide for interface toughness function

$$\Rightarrow \frac{G_0}{\Gamma_{IC}} = \frac{f(\psi)}{\left(1-\frac{\sigma_C}{\sigma}\right)\left(1+3\frac{\sigma_C}{\sigma}\right)}$$



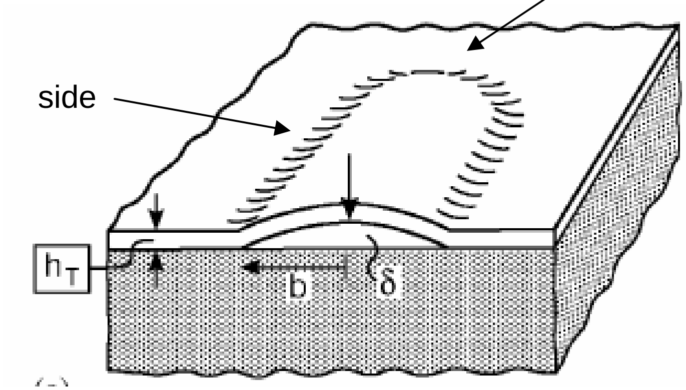
Stability of crack front requires: $\frac{\mathrm{d}}{\mathrm{db}}\left(\frac{G}{f(\psi)}\right) < 0.$

i.e. if tip "accidentally" advances, it is no longer critical.

Caution! This plot is difficult to interpret because each axis depends on σ

Inverse determination of interface toughness, stress (or modulus) by measuring buckling deflection and delamination width

<u>Straight-sided delamination without ridge crack on flat substrate</u>



$S \sim stretching\ stiffness$

$D \sim bending\ stiffness$

The basic results can be written as:

$$G_{SS} = \frac{1}{2} S \left(\frac{\pi}{4} \frac{\delta}{b} \right)^4$$

$$G_{side} = \frac{1}{2} S \left(\frac{\pi}{4} \frac{\delta}{b} \right)^4 + 2D \left(\frac{\pi}{b} \right)^2 \left(\frac{\pi}{4} \frac{\delta}{b} \right)^2$$

$$N_0 = D \left(\frac{\pi}{b} \right)^2 + S \left(\frac{\pi}{4} \frac{\delta}{b} \right)^2$$

Applies to any multilayer film with arbitrary stress distribution

If bending and stretching stiffness of the film are known, then the energy release rates and the resultant pre-stress can be determined by measurement of the deflection and the delamination width.

If resultant pre-stress is known, then the equations can be used to determine film modulus and release rates in terms of deflection and delamination width– see Faulhaber, et al (2006) for an example.

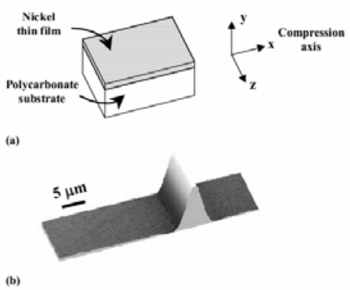
Metal or Ceramic Films on Compliant Substrates (Polymer or Elastomer)

Cotterell & Chen, 2000; Yu & Hutch, 2002; Parry, et al.,2005

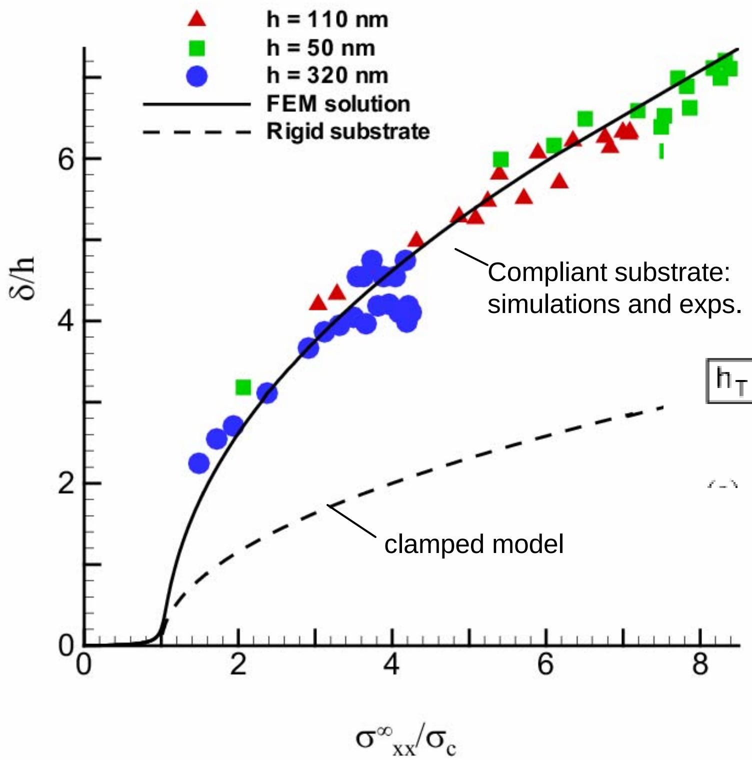
Analytical Fact: Edges of buckle delamination is effectively clamped if substrate modulus is larger than 1/3 of film modulus (i.e. clamped plate model is valid)

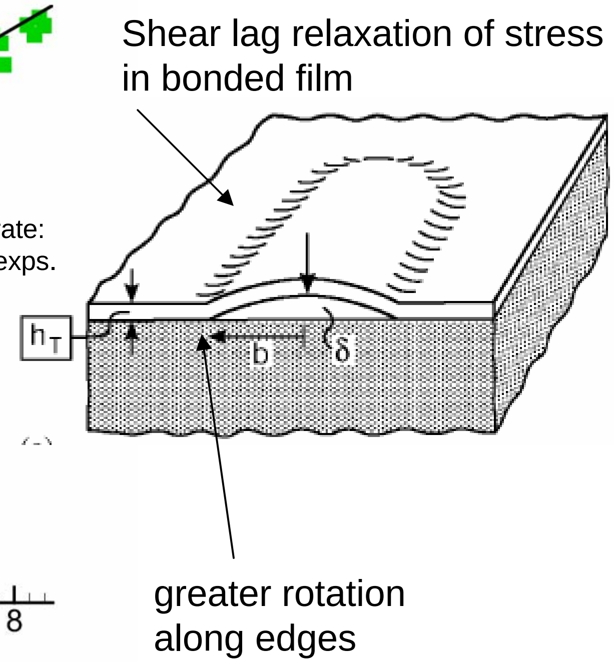
Highly compliant substrate has **three effects:**

1) Stabilizes straight-sided buckle delamination and tends to eliminate telephone cord morphology.
2) Significant film rotation occurs at edges of delamination and larger buckling deflections.
3) Relaxation of stress along bonded edges of delamination (shear lag effect) amplifies energy released.

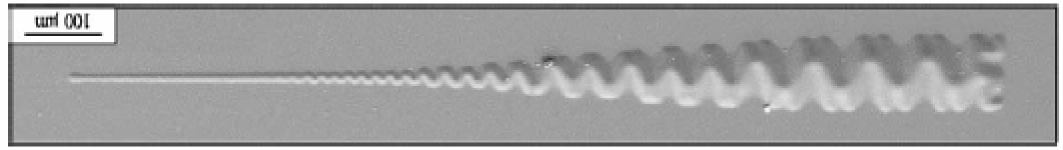


Ni films on polycarbonate substrates (Parry, et al.)



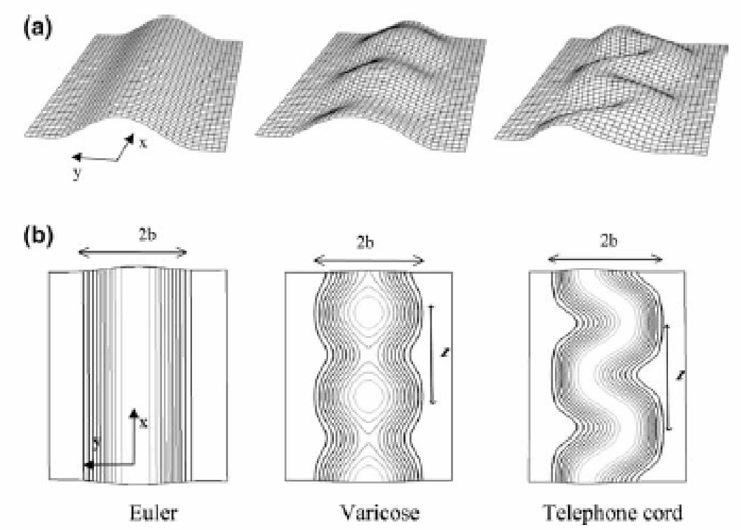


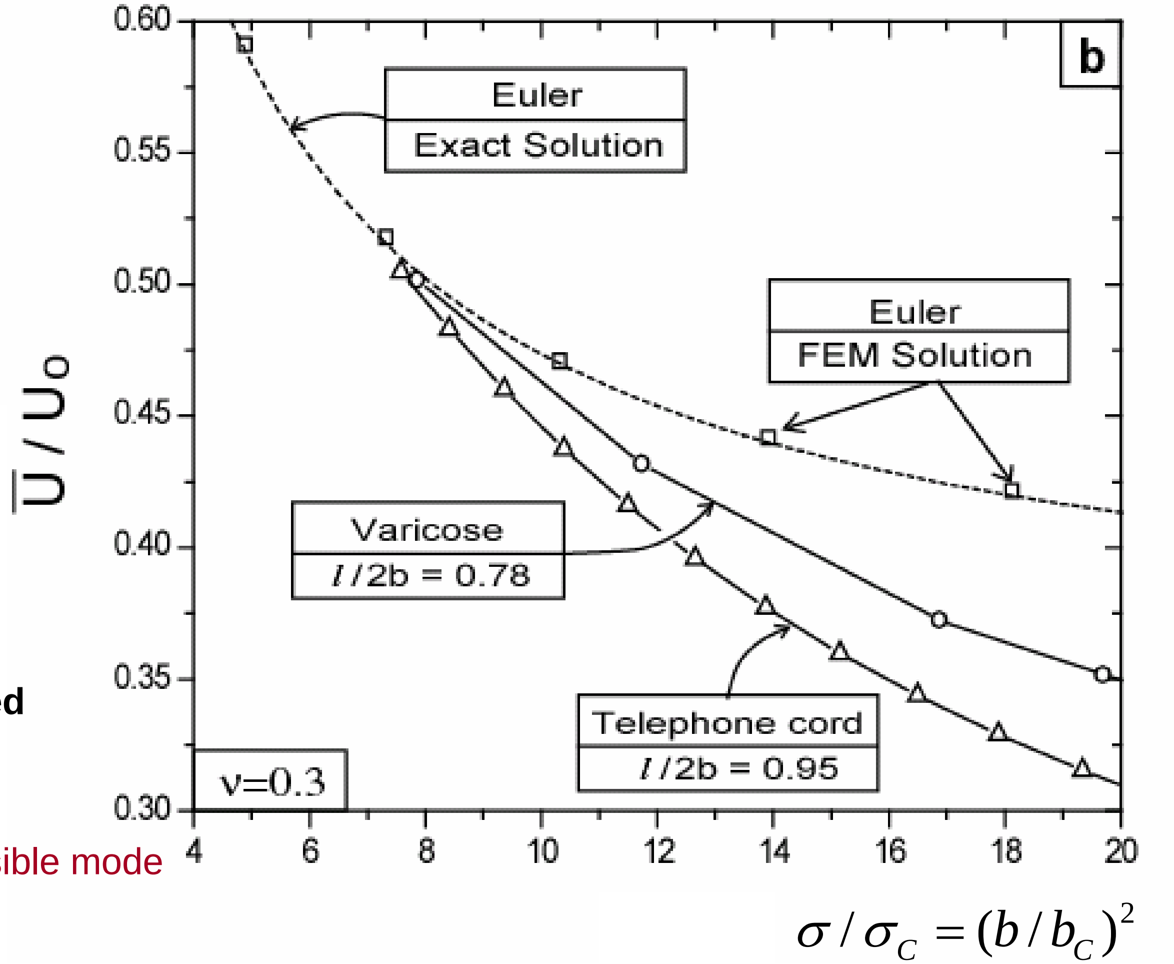
Energy Released as a Function of Morphology



DLC on silicon—tapered low adhesion interface: propagates from right to left

Three morphologies:





Film under equi-biaxial stress

Energy/area: $U_0 = \dfrac{\sigma^2 h}{E(1-\nu)}$

Energy/area in buckled film averaged over one full wavelength: $\bar{U}$

For $\sigma/\sigma_C < 6$:

Euler (straight-sides) mode is only possible mode

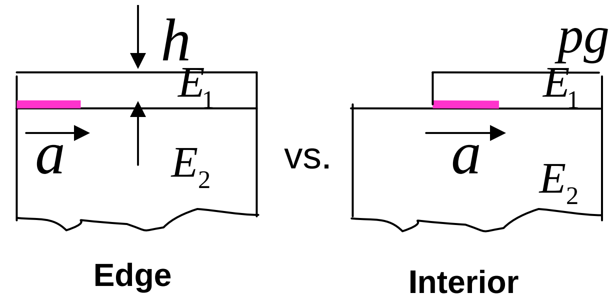
For $\sigma/\sigma_C > 7.5$:

Telephone cord morphology has lowest energy and releases the most energy/area.

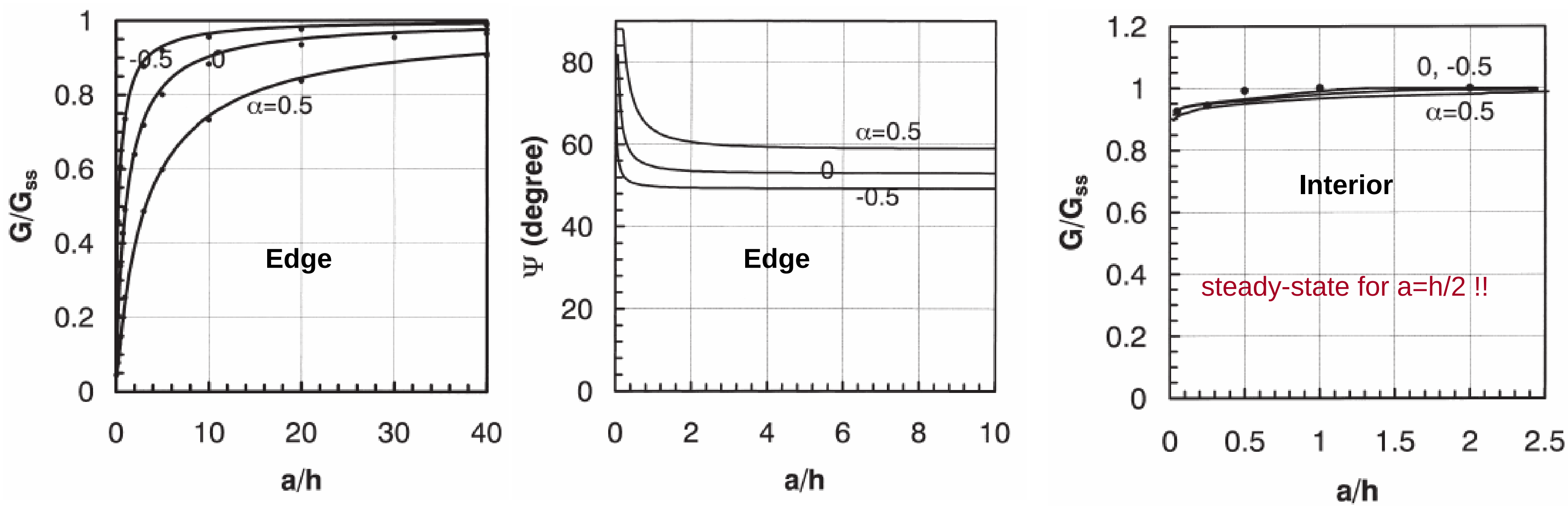
Real Time Propagation of a Telephone Cord buckle Delamination

M.-W. Moon

Delamination edge effects in plane strain

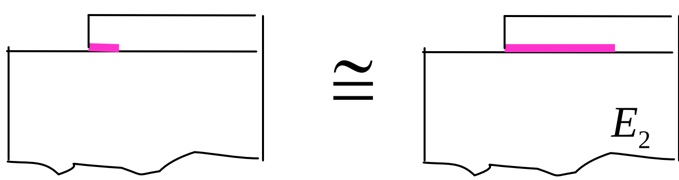


$$G_{SS} = \frac{1}{2}\frac{\sigma^2 h}{\overline{E}_1}$$

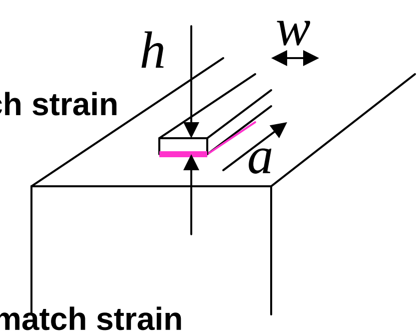


Conclusion: A compliant substrate (or even one with no mismatch) reduces the possibility of delamination initiating at the edge when a film extends to the edge of a substrate. The crack has to be ten times the film thickness, depending on the elastic mismatch, to attain steady-state.

If the film terminates in the interior of the substrate, there is no protection—the crack only has to be about ½ times the film thickness to reach steady state.



3D Effects for Delamination of Thin Film Strips



Energy/area stored in infinitely wide film subject to equi-biaxial thermal mismatch strain

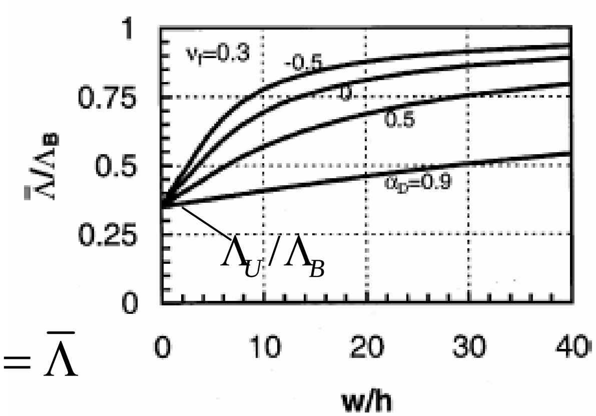
$$\Lambda_B = \frac{(1-\nu_1)\sigma^2 h}{E_1} = \frac{E_1}{(1-\nu_1)}\left(\Delta\alpha\Delta T\right)^2 h$$

Energy/area stored in a very narrow film strip subject to equi-biaxial thermal mismatch strain

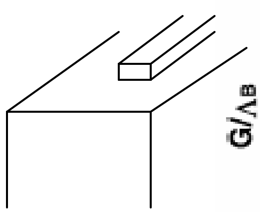
$$\Lambda_U = \frac{\sigma^2 h}{2E_1} = \frac{1}{2}E_1\left(\Delta\alpha\Delta T\right)^2 h$$

1D, no constraint perpendicular to strip

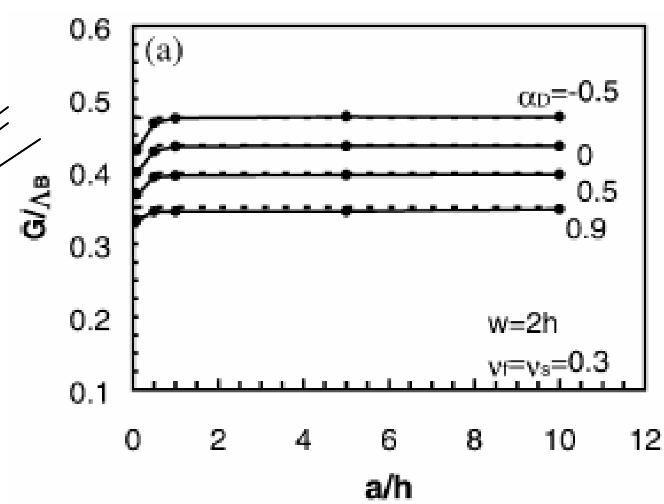
Average energy/area stored in infinitely long strip of width w subject to an equi-biaxial thermal mismatch strain. Even with no mismatch, the average energy is only 90% of that for an infinitely wide strip when w/h=50. If the substrate is very compliant the value may only slightly above the very narrow strip limit even when w/h=50.



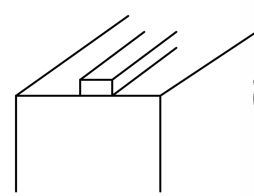
Steady-state energy release rate for strip delamination $a >> h$: $G_{SS} = \bar{\Lambda}$



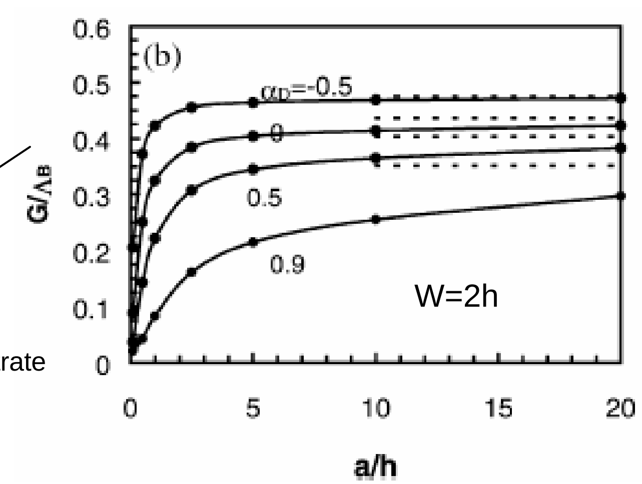
Interior end



versus



End at edge of substrate



DELAMINATION MECHANICS
Supplementary Notes and References

Page numbers refer to the slide page. A limited reference list is given on the last page.

Much of the mechanics outlined in the slides was developed around 1990 and is summarized in the article by Hutchinson and Suo (1992). Two other basic references with emphasis on interfaces are those by Evans and Hutchinson (1995) and Evans, Hutchinson and Wei (1995). For the student first getting acquainted with delamination mechanics, the notes, "Mechanics of thin films and multilayers", by Hutchinson (1996) cover some of the basic aspects in an assessable manner. It is assumed that the reader has a basic familiarity with fracture mechanics. Aspects of interface fracture mechanics are important in the developments, and if the author is not acquainted with this subject it might be good to start with Section II.C of Hutchinson and Suo (1990).

The slides also cover topics, in particular, extensions and applications, studied in the past few years. References are provided. It should be noted that the references listed on the last page are not intended to be comprehensive—they are primarily those of the author and his colleagues. These references will permit the reader access to other contributors and to the wider literature. The book on thin films by Freund and Suresh (2003) also provides excellent coverage of some delamination topics.

Page 1. This side provides a pictorial overview of the types of problems considered.

Page 2. The two-layer elasticity solution of Suo and Hutchinson (1990) for isotropic layers with differing moduli and Poisson's ratios has many applications. Dundurs' two dimensionless elastic mismatch parameters characterize the solution: in the slides they have been given for planes strain. Refer to the literature for plane stress definitions. The energy release rate can be obtained by simple methods simply by accounting for the difference between the energy well ahead and well behind the crack tip—see the notes by Hutchinson (1996). One reason for the usefulness and robustness of the energy release results from this solution derive elementary energy accounting. The relative proportion of mode II to mode I, as measured by ψ, requires a the full elasticity solution given by Suo and Hutchinson (1990). The examples in the slides are all based on the limiting case shown where the layer below the interface is very thick compared to the layer (or layers) above the interface, and the limit is for an infinitely deep layer below the interface.

If the second Dundurs mismatch parameter, β_D, is zero, the stresses in the singularity field characterizing the behavior near the tip of an interface have precisely the same form as in the homogeneous case with

$$\sigma_{\alpha\beta} = K_I \sqrt{\frac{1}{2\pi r}} f^I_{\alpha\beta}(\theta) + K_{II} \sqrt{\frac{1}{2\pi r}} f^{II}_{\alpha\beta}(\theta)$$

where r and θ are planar polar coordinated centered at the tip. The functions f^I and f^{II} are the same as those for the homogeneous material. If β_D is not zero, the stresses associated with the crack tip singularity are more complicated—a so-called oscillatory singularity. In all the examples considered in the slides we will take $\beta_D = 0$ since this captures most of the essential features of the phenomena of interest. Students interested in pursuing the effect of non-zero β_D can start off by looking at Section II.C.

Page 3. This is the basic result which will be used throughout the slides.

Page 4. To see the effect of friction on the mode II edge delamination crack see the reference by Balint and Hutchinson (2001).

Page5. Results for $\omega(\alpha_D)$ for α_D near unity (i.e. for stiff films on very compliant substrates such as metals on polymers) have not been published and do not appear to be available.

Page 6. See Evans and Hutchinson (1995) and Evans, Hutchinson and Wei (1999) for discussion of interface toughness and other systems.

Page 8. Reference on delamination in presence of temperature and stress gradients: Evans and Hutchinson (2006).

Page 9. The basic solution for an isolated crack in a homogeneous material subject to a temperature gradient was given by Sih (1962). See Hutchinson and Lu (1995), Hutchinson and Evans (2002) and Evans and Hutchinson (2006) for work specifically related to temperature gradients and their role in delamination of coatings.

Page 10. Reference: Evans and Hutchinson (2006).

Page 11-13. There is now a large literature on buckling delamination covering both theoretical and experimental aspects. Basic mechanics covered in the slides is given in Hutchinson and Suo (1992), Section VI, and Hutchinson, Thouless and Liniger (1992).

More recent references are Moon et al. (2002) and Moon et al (2004); additional references are cited in these papers.

Page 14. This approach has been developed in Faulhaber et al. (2006) with application to delamination of thermal barrier coatings on curved substrates. The approach has also been extended in this paper to delaminations with ridge cracks.

Page 15. Theoretical and experimental work for straight-sided buckle delaminations for stiff films on polymeric substrates have been published in Cotterell & Chen (2000) Yu & Hutchinson (2002); Parry et al. (2005).

Page16. The reference for this slide is Moon et al. (2004).

Page 17. The movie of the real time evolution of a buckle delamination was supplied by M-Y. Moon. See the work of A. Volinsky for many interesting examples of buckle delamination.

Page 18. These and other related results are given by Yu, He and Hutchinson (2001).

Page 19. Three-dimensional results for delamination of thin film strips are presented in Yu and Hutchinson (2003).

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